1. (25%) Solve

$$y_1' = 3y_1 + 3y_2 + 8$$
$$y_2' = y_1 + 5y_2 + 4e^{3t}$$

by using matrix method.

2. (25%) Solve

$$y'' + y' + 9y = 0, \quad y(0) = 0.16, \quad y'(0) = 0 \quad \text{by using Laplace Transform.}$$

Note: A table for some functions $f(t)$ and their Laplace Transforms $\mathcal{L}(f)$.

| | $f(t)$ | $\mathcal{L}(f)$ | | $f(t)$ | $\mathcal{L}(f)$ |
|---|---|---|---|---|---|
| 1 | 1 | $1/s$ | 7 | $\cos \omega t$ | $\dfrac{s}{s^2 + \omega^2}$ |
| 2 | $t$ | $1/s^2$ | 8 | $\sin \omega t$ | $\dfrac{\omega}{s^2 + \omega^2}$ |
| 3 | $t^2$ | $2!/s^3$ | 9 | $\cosh at$ | $\dfrac{s}{s^2 - a^2}$ |
| 4 | $t^n$ $(n = 0, 1, \cdots)$ | $\dfrac{n!}{s^{n+1}}$ | 10 | $\sinh at$ | $\dfrac{a}{s^2 - a^2}$ |
| 5 | $t^a$ ($a$ positive) | $\dfrac{\Gamma(a+1)}{s^{a+1}}$ | 11 | $e^{at} \cos \omega t$ | $\dfrac{s-a}{(s-a)^2 + \omega^2}$ |
| 6 | $e^{at}$ | $\dfrac{1}{s-a}$ | 12 | $e^{at} \sin \omega t$ | $\dfrac{\omega}{(s-a)^2 + \omega^2}$ |

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$$
$$\mathcal{L}(f'') = s^2\mathcal{L}(f') - sf(0) - f'(0)$$

3. (25%) Find the Fourier series of the function

$$f(x) = \begin{cases} 0, & if -2 < x < -1 \\ k, & if -1 < x < 1 \\ 0, & if \ 1 < x < 2 \end{cases}$$

4. (25%) Solve the following initial value problems.

$$(u_x)^2 \cdot u_y = 1, \ u(x, 0) = 3x + 1.$$