淡江大學 105 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　　科目：機率與統計學　　　18－1

考試日期：7 月 22 日(星期五) 第 1 節　　　本試題共 5 大題， 1 頁

1. **(25%)** Let the joint p.m.f. of X and Y be

$f(x,y) = 1/4,\quad (x,y) \in S = \{(0,0),(1,1),(1,-1),(2,0)\}$.

i). Compute $E(X)$ and $\mathrm{Var}(X)$

ii). Compute $E(X \mid Y=0)$ and $Var(X \mid Y=0)$.

iii) Find the covariance of X and Y.

iv) Prove or disprove that X and Y are independent.

v). Show that $E(X) = E(E(X \mid Y))$

2. **(20%)** Customers arrive in a certain shop according to Poisson process at mean rate of 20 per hour (λ=20 persons/hr). Let T denote the waiting time in minutes until the first customer arrival.

i). Find the p.d.f. and the moment generating function of T.

ii). What are the mean and variance of T?

iii). Show that $P(T > 5 \mid T > 3) = P(T > 2)$.

3. **(20%)** Let X_1, X_2, X_3 and X_4 be random variables from $U(0,\theta)$, and denote their order statistics by $X_{(1)}$, $X_{(2)}$, $X_{(3)}$ and $X_{(4)}$, respectively.

i). Show that the largest order statistic $X_{(4)}$ is sufficient for θ.

ii). Compute the probability $F(t) = P(X_{(4)} \le t)$.

iii). Compute $E(X_{(4)})$.

iv). Find a function $h(X_{(4)})$ so that $Eh(X_{(4)}) = \theta$.

4. **(15%)** Let X_1, X_2 and X_3 be i.i.d. random variables from $N(\mu, 12)$. Consider the test for the null hypothesis $H_0 : \mu = 0$ against the alternative hypothesis $H_1 : \mu = 1$ with significance level 0.05.

i). Construct a critical region A so that the test has level 0.05.

ii). Find the power of this test.

iii). Compute the p-value if we observe $X_1 = 1, X_2 = 0.96$ and $X_3 = 1.96$.

5. **(20%)** In a simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i,\quad i = 1, \cdots, n$, where ϵ_i are i.i.d. $N(0,\sigma^2)$.

i). Find the maximum likelihood estimates (mle) $\hat{\alpha}, \hat{\beta}$ and $\hat{\sigma}^2$ of α and β and σ^2, respectively.

ii). Show that mle of β is unbiased.

iii). Explain why $\hat{\beta}$ and $\hat{\sigma}^2$ are independent.

iv). Find a 95% confidence interval for β when σ^2 is unknown.