# 淡江大學 105 學年度日間部轉學生招生考試試題

系別：數學學系三年級　　　　科目：線性代數　　　　17-1

考試日期：7 月 22 日(星期五) 第 1 節　　　　本試題共 6 大題， 1 頁

**Please show all your work to receive full credit.**

1. (20 pts) Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 3 & 7 & 1 & 4 \\ 1 & 4 & 2 & 3 \end{bmatrix}$.

(a) Find a basis for the column space of $A$.

(b) Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. If the linear system $A\mathbf{x} = \mathbf{b}$ has a solution, what condition on $b_1, b_2, b_3$ should be satisfied?

2. (15 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Suppose $T(1,2) = (3,4)$ and $T(3,4) = (1,2)$. Find a formula for $T(x,y)$, where $x, y \in \mathbb{R}$.

3. (20 pts) Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. (Note that $A$ is a positive definite symmetric matrix.)

(a) Verify that $\lambda = 1$ and $\lambda = 4$ are eigenvalues of $A$.

(b) For each eigenvalue of $A$, find an orthonormal basis for the associated eigenspace.

(c) Find a square matrix $B$ such that $B^T B = A$.

4. (15 pts) Let $P_2 = \{a_0 + a_1 x + a_2 x^2 \mid a_0, a_1, a_2 \in \mathbb{R}\}$, which is a vector space over $\mathbb{R}$.

(a) Verify that $B = \{1 + x + x^2, 1 + x^2, 2 + x + 4x^2\}$ is a basis for $P_2$.

(b) Find the coordinate vector of $1 + 2x + 3x^2$ with respect to $B$.

5. (15 pts) Suppose $v_1, v_2, v_3$ are all nonzero vectors in $\mathbb{R}^3$. If $S = \{v_1, v_2, v_3\}$ is an orthogonal set, show that $S$ forms a basis for $\mathbb{R}^3$.

6. (15 pts) Suppose $A$ is an $n \times n$ matrix with real entries. If $\lambda = 0$ is an eigenvalue of $A$, show that $A$ is not invertible.