

淡江大學 105 學年度日間部轉學生招生考試試題

系別：數學學系二年級

科目：微積分

6-1

考試日期：7月21日(星期四) 第1節

本試題共 6 大題， 1 頁

1. Evaluate the values (每題 5 分, 共 20 分).

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2}$.

(b) $\lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{e^x - 1} \right]$.

(c) $\lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+(n-1)} + \frac{1}{n+n} \right)$.

(d) Suppose $f : (-2, 2) \rightarrow \mathbb{R}$ is a differentiable function, $f(1) = 1$ and $f'(1) = 2$.

Evaluate $\lim_{x \rightarrow \pi/4} \frac{f(\tan(x)) - 1}{x - \pi/4}$.

2. Test for convergence or divergence (每題 6 分, 共 12 分).

(a) $\int_0^{\infty} e^{-x^2} dx$.

(b) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

3. Evaluate the integrals (每題 8 分, 共 32 分).

(a) $\int_0^1 \cos^2 x \sin x dx$.

(b) $\int_0^{\pi/2} x \sin x dx$.

(c) $\iint_{\Omega} \cos\left(\frac{x-y}{x+y}\right) dA$, where Ω is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.

(d) $\iiint_B e^{4(x^2+y^2+z^2)^{3/2}} dV$, where $B = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$.

4. Use $\delta - \epsilon$ argument to prove $\lim_{x \rightarrow 3} x^2 = 9$. (12 分)

5. Let $f(x) = e^x$, for $x \in \mathbb{R}$.

(a) Find the Maclaurin series for $f(x) = e^x$. (4 分)

(b) Show the Maclaurin series of f and f' are identical for all $x \in \mathbb{R}$. (8 分)

6. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest and farthest from the origin. (12 分)