淡江大學 101 學年度碩士班招生考試試題

系別：財金系、國企系
產經系、經濟系

科目：統　計　學

考試日期:2 月 26 日(星期日) 第 3 節

本試題共 8 大題，4 頁

Do the problems in the order they are given. Show all work and <u>circle your answer.</u>

(10 points)

1. A *<u>fair</u>* coin is tossed 20 times with S = the number of heads. Compute the value of β which defined as the following ratio of probabilities

$$\beta = \frac{\Pr[S = 14]}{\Pr[S = 17]}$$

(10 points)

2. Suppose that $Z = \alpha X + \beta Y$, where α and β are *<u>any</u>* real numbers and where

$$E[X] = 5,\ E[Y] = 10,\ var[X] = 1,\ var[Y] = 4,\ corr[X,Y] = -\frac{1}{2}$$

What values of α and β minimize $var[Z]$ subject to the constraint $E[Z] = 10$.

(10 points)

3. Suppose that X is a random variable with probability density $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and zero elsewhere. Calculate the following ratio for the random variable X

$$\eta = \frac{mean}{median}$$

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(15 point)

4. Suppose that $Y_t = \beta X_t + \varepsilon_t$, for t = 1,…,N and where $E[\varepsilon_t] = 0, var[\varepsilon_t] = \sigma^2$,

$cov(\varepsilon_t, \varepsilon_s) = 0$ *for* $t \neq s$, and $cov(\varepsilon_t, X_s) = 0$ for all t and s.

(5 points) (i) Calculate the variance of the estimator $\hat{\beta}' = \frac{\overline{Y}}{\overline{X}}$

(5 points) (ii) Calculate the variance of the estimator $\hat{\beta} = \frac{\sum X_t Y_t}{\sum X_t^2}$

(5 points) (iii) Show which is smaller (i) or (ii) ?

(15 points)

5. A large school claimed that at least 20% of its students are female. To test this, a teacher sampled 400 students at random. The teacher claimed that he would reject the school's claim if the sample had less than or equal to 70 women. Assuming the true proportion is 20%, what is the p-value of his test?

(10 points)

6. Calculate each of the following assuming $Pr[B] = \frac{1}{3}$ and $Pr[A \cap B] = \frac{1}{4}$ and $Pr[A \cup B] = 1$ with B^c = the complement of B (i.e. $B^c \cap B = \varnothing$)

(2 points) a. $Pr[B \mid A]$ = _____

(2 points) b. $Pr[A \cup B^c]$ = _____

(2 points) c. $Pr[(A^c \cap B) \cup (A \cap B^c]$ = _____

(2 points) d. $Pr[A^c \cap B^c]$ = _____

(2 points) e. $Pr[A \cap B \mid B]$ = _____

(15 points)

7. The following data are independent samples taken on random variables X and Y, which are known to have distributions $N(\mu_X,1)$ and $N(\mu_Y,2)$, respectively. Note that the variances of X and Y are known exactly and are different.

X	-0.13	1.37	0.28	2.21	-0.37	-0.60	-0.85	2.30	-0.33	0.38	0.74	0.80
Y	-3.24	1.96	-2.23	-0.46	0.92	-0.37	0.85	0.42	0.97	-0.57	0.68	-1.19

Use the standard normal table below to test the hypothesis $H_o : \mu_X = \mu_Y$ against the alternative $H_a : \mu_X \neq \mu_Y$ assuming $\alpha = 0.05$ as the significance level, and answer the following questions.

(5 points) (i) What is the exact formula for the test statistic?

(5 points) (ii) What is the sample value of this test statistic?

(5 points) (ii) Do you reject H_o for this value of the test statistic? Why?

(15 points)

8. The following data are independent samples (rounded for convenience) on random variables X and Y, which are known to have distributions $N(3,\sigma_X^2)$ and $N(3,\sigma_Y^2)$, respectively. Note that the means of X and Y are known exactly and are equal to 3.

X	3	3	5	4	1	1	3	2	2	5	3	4
Y	2	5	5	1	1	3	1	5	6	3	2	4

Use the F distribution table below to conduct a two tailed test of the hypothesis $H_o : \sigma_X = \sigma_Y$ against the alternative $H_a : \sigma_X \neq \sigma_Y$ assuming an $\alpha = 0.05$ significance level and then answer the following questions.

(5 points) (i) What is the exact formula for the test statistic?

(5 points) (ii) What is the sample value of this test statistic?

(5 points) (iii) Do you reject H_o for this value of the test statistic? Why?

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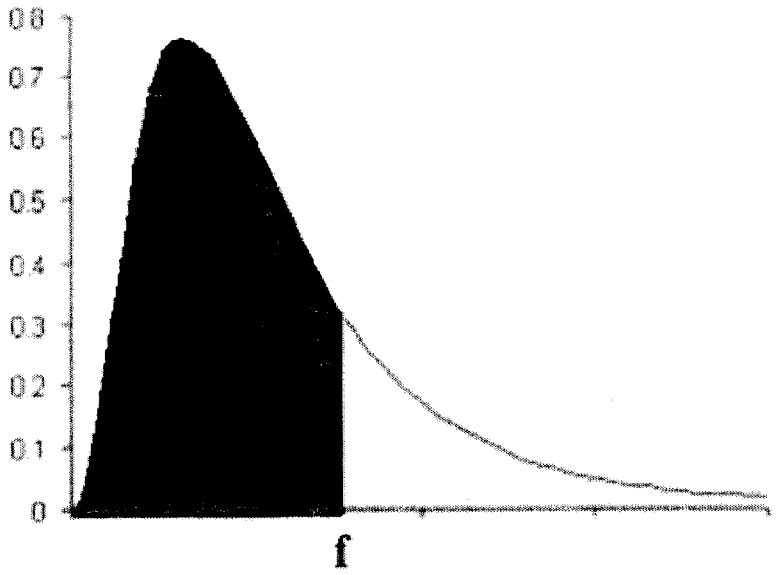
Standard Normal Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9031	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

Approximate % Area to the Left of F(12,12) Distribution



f	0.00	0.30	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.00	1.50	2.0	2.5	3.0	3.50	4.0	4.5
%	0.00	0.025	0.04	0.09	0.16	0.23	0.31	0.39	0.47	0.50	0.75	0.88	0.94	0.975	0.98	0.99	0.99