淡江大學99學年度碩士班招生考試試題

系別：財金系、保險系、國企系
產經系、經濟系

科目：統　計　學

准帶項目請打「V」	
	計 算 機

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本試題雙面印製

依題號順序作答，答案數字以分數或取至小數點下兩位表示。

I. (60%, 6 points each)

1. A fair coin is to be tossed until the first head appears. What are the chances of that happening on an even-numbered toss?

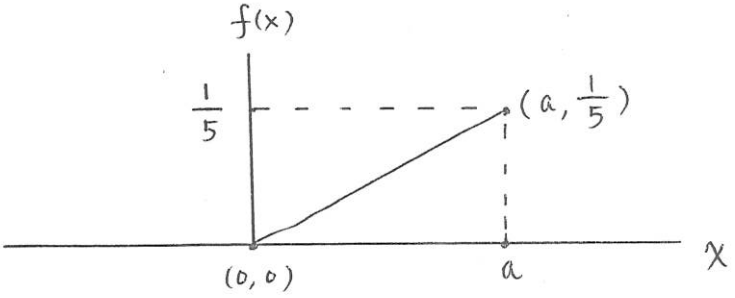
2. Two numbers are chosen independently and at random from the interval (0, 1). What is the probability that the two numbers differ by more than 0.5?

3. An urn contains nine chips, four white and five blue. Two are drawn out at random without replacement. Let X be the number of blue chips in the sample. Find E(X).

4. Let X be a continuous random variable with the density function

$$f(x)=\begin{cases}2e^{2x-3}, x<3/2\\ 0, otherwise\end{cases}$$, what is the 60th percentile of X?

5. Let X be a continuous random variable with the density function $f(x)$ as shown below:



What is the 20th percentile of the distribution of X?

6. Let A and B be independent events, $P(A\cap B)=\frac{1}{10}$, and $P(A\cap B')=\frac{1}{5}$, where B' represent the complement of B, evaluate $P[(A\cup B)']$.

7. Let A and B be events such that $P(A)=P(B)=0.5$ and $P(A'\cap B')=\frac{1}{3}$, where A' and B' represent the complement of A and B, respectively.

Evaluate $P(A'\cup B)$.

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8. Let X and Y be two random variables whose joint pdf was given as follows:

		Y		
	0	1	2	3
X				
0	1/8	1/4	1/8	0
1	0	1/8	1/4	1/8

Define f(x, y) = 3x – 2xy + y. Find E[f(x, y)].

9. Let $Y_1,\ldots,Y_n$ be random variables such that $E(Y_i) = 2 + \beta x_i$, $i = 1,\ldots,n$. Given observations $(x_1, y_1),\ldots,(x_n, y_n)$, find the least squares estimator for β.

10. The hypothesis $H_0 : \mu = 0$ is tested against $H_1 : \mu = 1$ using a one-sample t-statistic when sampling from a normal distribution. If $\alpha = P$ (Type I error) and $\beta = P$ (Type II error), which of the following statements are true?

(a) If the sample size n is fixed and α is increased, then β decreases.
(b) If α is held constant and n is increased, then β decreases.
(c) $\alpha + \beta = 1$

II. (40%, 5 points for each of the eight subproblems)

Department store **Sogo** sells personal computers (PCs) as well as printers. The number of computers and printers sold varies from day to day, but the store manager obtained the sales history over the past 200 days in the form of following table:

The frequency distribution of two random variables, X and Y.

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	Number of Printers Sold (Y)				
	0	1	2	3	4
Number of PCs Sold (X)					
0	6	4	2	2	2
1	6	10	4	2	2
2	4	12	20	10	2
3	4	4	10	20	10
4	2	2	10	20	30

In this example we have two random variables, X and Y. This table shows that of the 200 days, there were 30 days when the store sold 4 PCs and 4 printers, but on two days, although it sold 4 PCs, it sold no printers. Other entries in the table are to be interpreted similarly.

(1) Find the marginal (i.e., unconditional) distributions of X and Y, namely, f(X) and f(Y).

(2) Find the conditional probability f(X = 2|Y= 0) and f(Y = 4 |X = 3).

(3) What is the average number of PCs sold per day? And What is the average number of printers sold per day?

(4) Compute the (unconditional) variance of 5X, var(5X).

(5) Compute the covariance $\text{cov}(2X-1,\ 2-Y)$ between the two random variables, $2X-1$ and $2-Y$.

(6) Compute the coefficient of correlation ρ between X and Y.

(7) Compute E(Y| X = 3).

(8) Compute the conditional variance of Y given that X is 3, Var(Y | X = 3).