淡江大學105學年度碩士班招生考試試題

28-1

系別：電機工程學系通訊與電波組　　科目：工程數學

考試日期：3月5日(星期六) 第2節　　本試題共 6 大題， 1 頁

(1) (15%) Find the derivate of $f(x)=3^{\frac{\ln(x+2)}{\ln(x-2)}}$, and then evaluate it at x=2.

(2) (15%) (i) Assume $r=(x^2+y^2+z^2)^{\frac{1}{2}}$; please prove that : $rdr=xdx+ydy+zdz$

(ii) Assume the function $\vec{F}$ is $\vec{F}=\hat{x}\frac{\partial\varphi}{\partial x}+\hat{y}\frac{\partial\varphi}{\partial y}+\hat{z}\frac{\partial\varphi}{\partial z}$; please prove that: $\nabla\times\vec{F}=0$

Hint: $\nabla\times\vec{A}=\hat{x}(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z})+\hat{y}(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x})+\hat{z}(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y})$

(3) (15%) For the following equation, find all the different forms of the solution:

$y''(x)+k^2y(x)=0$

Hint: Consider the cases of $k^2>0$, $=0$, and <0, respectively.

(4) (15%) When we try to use the series function (1) to approximate the function f(x) (2)

$g(x)=\sum_{n=1}^{\infty}C_n\sin(\frac{n\pi}{b}x)$ ...(1)

$f(x)=\begin{cases}-1, -b<x<0\\ +1, 0<x<b\end{cases}$ ...(2)

(i) find the coefficients C_n ; (ii) What are the values of g(x) outside [-b, b] ?

(5) (20%) Assume the function $\vec{F}$ is $\vec{F}=\frac{\hat{x}x+\hat{y}y+\hat{z}z}{(x^2+y^2+z^2)^{\frac{3}{2}}}$ and let points $P_1=(1,0,0)$、$P_2=(2,0,0)$;

i) Please find the value of the line integral $\int_{P_1}^{P_2}\vec{F}\cdot\overrightarrow{dl}$ along the line segment $\overline{P_1P_2}$.

ii) Let points $P_3=(0,0,1)$, $P_4=(0,0,2)$, Please find the value of the line integral $\int_{P_3}^{P_4}\vec{F}\cdot\overrightarrow{dl}$ along the line segment $\overline{P_3P_4}$.

(6) (20%) For the following equation,

$\frac{d^2y}{dt^2}=y^{-\frac{1}{2}}$

with the initial conditions $y=0$ & $\frac{dy}{dt}=0$ at $t=0$; please solve this for $y(t)$.

Hint: you may start by multiplying both sides of the equation with $2\frac{dy}{dt}$.