淡江大學 105 學年度碩士班招生考試試題

15-1

系別：數學學系B組　　　　科目：統計學

考試日期：3月5日(星期六) 第3節　　　　本試題共 6 大題，1 頁

1.(20%). Suppose that the distribution for blood type is 50% of "O", 30% of "A", 15% of "B" and 5% of "AB". A random sample contains 20 persons and let X_O, X_A, X_B and X_{AB} denote the number of persons of blood type "O", "A", "B" and "AB", respectively.

i). What is the distribution of X_O.

ii). What is the probability $P(X_O = 8, X_A = 7, X_B = 4, X_{AB} = 1)$

iii). Compute $E(X_O X_A)$

2.(20%) Consider the simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i, \quad i = 1, \cdots, n$, where ϵ_i are i.i.d. $N(0, \sigma^2)$, let $\hat{\alpha}$, $\hat{\beta}$ be the least squares estimates of α and β.

i). Write down the likelihood function.

ii). Prove that $\hat{\beta} = S_{XY}/S_{XX}$, $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, where $S_{XY} = \Sigma[(Y_i - \bar{Y})(X_i - \bar{X})]$, and $S_{XX} = \Sigma(X_i - \bar{X})^2$.

iii). Compute the M.L.E. for σ^2

iv). Show that $\hat{\beta}$ and $\hat{\alpha}$ are unbiased estimators of β and α.

3.(10%) Let p equal the proportion of Americans who favor the death penalty. If a random sample $n = 1000$ Americans yield $y = 800$ who favored the death penalty, find an approximate 90% confidence interval for p.

4.(10%) Customers arrive in a certain shop according to an approximate Poisson process at a mean rate of 0.8 person/minute. Let X denote the waiting time in minutes until the first customer arrives.

i). Find the cdf $F(t) = P(X \leq t)$ of X.

ii). Show that $\lambda(t) = \frac{F'(t)}{S(t)}$ is a constant where $S(t) = 1 - F(t)$.

5. (20%) Let $X_{(1)}, \cdots, X_{(4)}$ be the order statistics of $X_1, \cdots, X_4$, where $X_1, \cdots, X_4$ are i.i.d. from $U(0, \theta)$.

i). Show that $X_{(4)}$ are the maximum likelihood estimate of θ.

ii).What is the conditional distribution of X_1 given $X_{(4)} = 2$? Is $X_{(4)}$ sufficient for θ?

iii). Compute $E[X_{(4)}]$, than find an unbiased estimator of θ.

6.(20%) Let $X_1, \cdots, X_{25}$ be iid $N(\mu, 100)$. To test $H_0 : \mu = 80$ against $H_1 : \mu > 80$, let the critical region be defined by $C = \{(x_1, x_2, \cdots, x_{25}) : \bar{x} \geq 83.3\}$.

a). What is the power function $\beta(\mu)$ for this test ? Sketch it.

b). What is the significance level of this test ?

c) What is the p-value corresponding to $\bar{x} = 82$.

d). This test is known to be uniformly most powerful. Why?