1. (12 pts) Let $A = \begin{bmatrix} 2 & 0 & 1 & 6 \\ 0 & 3 & 0 & 5 \\ 14 & -15 & 7 & 17 \end{bmatrix}$.

Find the rank and nullity of A. Please show your work.

2. (10 pts) Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $(1, -2, 0, 1), (-2, 1, 0, 1)$, $(2, -2, 0, 0), (1, 3, 0, 2)$.

3. (10 pts) Let $T : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation such that $T(1, 0, 0) = 1$, $T(1, 1, 0) = 2$, $T(1, 1, 1) = 3$. Determine the value $T(a, b, c)$ for $a, b, c \in \mathbb{R}$.

4. (20 pts) Let $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

(a) Find the eigenvalues and the corresponding eigenspaces for the matrix A.

(b) Find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.

5. (12 pts) Give an example of a square matrix A such that the eigenvalues of A are all real numbers, but A is not diagonalizable. Please justify your answer.

6. (12 pts) Let $V = \{(x, y) \,|\, xy \geq 0\}$. Determine if V is a subspace of $\mathbb{R}^2$. Please explain your reason.

7. (12 pts) Suppose A and B are $n \times n$ similar matrices.

(a) Is it true that A and B must have the same eigenvalues? Why?

(b) Is it true that A and B must have the same eigenvectors? Why?

8. (12 pts) Let A be an $n \times n$ matrix with $\det(A) = 1$. If all the entries in A are integers, show that all the entries in A^{-1} are also integers.