淡江大學 105 學年度碩士班招生考試試題

系別：數學學系　　　　科目：微積分　　　13-1

考試日期: 3 月 5 日(星期六) 第 2 節　　　本試題共 6 大題, 1 頁

(1) (10 points) Evaluate the following limits

(a) $\lim_{n\to\infty}(\frac{1}{n+3}+\frac{1}{n+3\cdot 2}+\cdots+\frac{1}{n+\ 3k}+\cdots\frac{1}{n+3n})$.

(b) $\lim_{h\to 0}\frac{\int_0^3 \sin(x)e^x\,dx-\int_0^{3+h}\sin(x)e^x\,dx}{h}$

(2) Find the equation of tangent line to the curve at the given point below.

(a) (7 points) $x^2+2y+e^y=2$ at $(1,0)$.

(b) (8 points) $y=f^{-1}(x)$ at $(e,1)$ where $f(x)=xe^x$, for $x>0$.

(3) (18 points) Test for convergence or divergence

(a) $\int_2^\infty \frac{1}{x\ln x}\,dx$

(b) $\sum_{n=1}^\infty \frac{n^{1/2}}{n^2+2n+3}$.

(c) $\sum_{n=1}^\infty \frac{\ln n}{n^{3/2}}$.

(4) (32 points) Evaluate the integrals

(a) $\int_0^{\pi/2}\cos^2 x\sin x\,dx$,

(b) $\int_0^\infty e^x\cos x\,dx$,

(c) $\int_0^\infty e^{-x^2}\,dx$,

(d) $\int_0^1\int_x^1 e^{y^2}\,dydx$.

(5) (15 points) Let K be a compact set in $\mathbb{R}^n$. Prove that if $f:K\to K$ is a contraction mapping then f has a unique fixed point in K.

(6) (10 points) $-\infty<a<b<\infty$. Suppose $f:(a,b)\to\mathbb{R}$ is uniformly continuous. Prove that there is a continuous function g defined on $[a,b]$ so that $g(x)=f(x)$ for all $x\in(a,b)$.