

## 淡江大學 102 學年度碩士班招生考試試題

系別：數學學系

科目：基礎數學（含微積分、線性代數）

考試日期：3 月 10 日(星期日) 第 2 節

本試題共 6 大題， 1 頁

1. (35%) Find (a)  $\lim_{x \rightarrow 4} \frac{x-4}{x^2-8x+16}$  (b)  $\frac{d}{dx} \left( \left[ \frac{3 \sin x}{x+e^{2x}} \right]^2 \right)$  (c)  $\frac{dy}{dx} \Big|_{(x,y)=(3,1)}$  if  $x^2 + 5y^3 = xy^2 + 11$ .

(d)  $\int x^3 e^{x^4+2} dx$  (e)  $\int_1^4 \frac{1}{x^2} dx$

2. (10%) Find the volume of the solid below  $z = e^{x+y}$  and above the region

$$D = \{(x, y) \mid x \geq 0, y \geq 0, x + y \leq 1\}.$$

3. (15%) Find all global extrema of  $f(x, y) = -x^3 + 9x - 4y^2$  on  $S = \{(x, y) \mid x \geq 0, y \geq 0, x - y \leq 2\}$ .

4. (10%) Let  $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ . Find the inverse of  $A$  and the determinant of  $(3A)^{-1}$

5. (12%) Define  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$  as  $T(x, y, z) = (2x - 2y, 3z)$ .

(a) Find the range, its basis, and rank of  $T$ .

(b) Find the null space, its basis, and the nullity of  $T$ .

6. (18%) Let  $A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & -1 & 4 \\ -2 & 4 & -1 \end{bmatrix}$ . Find an orthogonal matrix  $P$  such that  $P^T A P$  is a diagonal

matrix.