25-1

系別：數學學系　　　　　　科目：微積分 60% 及線性代數 40%

准帶項目請打「V」

	簡單型計算機

本試題共 ___1___ 頁

請詳列演算過程, 否則不予計分, 每題 10 分, 共 10 題

1. Find the interval of convergence of the series. $\displaystyle\sum_{n=0}^{\infty}\frac{(-2)^n(x+1)^n}{\sqrt{n+1}}$.

2. Determine whether the improper integral $\displaystyle\int_0^{\infty}\frac{dx}{\sqrt{x}(x+2)}$ is convergent or divergent ?

3. Evaluate the given integral $\displaystyle\int\cos(\ln x)dx$.

4. Sketch the curve $y=\dfrac{2+x-x^2}{(x-1)^2}$

5. Find the area of the region R bounded by the curves
   $xy=1,\quad xy=3,\quad xy^{1.4}=1,\quad xy^{1.4}=2$.

6. Find the volume of the solid that is bounded by the surface $z=y\sqrt{1+x^3}$ and over the region bounded by x- axis , y=x and x=2.

7. Compute the rank of $A=\begin{bmatrix}1 & -2 & 1 & 1\\ -1 & 2 & 0 & 1\\ 2 & -4 & 1 & 0\end{bmatrix}$ and find bases for the row space and the column space of A.

8. Let $A=\begin{bmatrix}5 & -3 & -2\\ 8 & -5 & -4\\ -4 & 3 & 3\end{bmatrix}$. Find a matrix P such that $P^{-1}AP$ is a diagonal or upper triangular matrix .

9. Let P_2 denote the set of polynomials with degree less or equal to 2. Let

   $\langle p,\quad q\rangle=\displaystyle\int_0^{\infty}p(x)q(x)e^{-x}dx$, where p(x), q(x) are polynomials, be a inner product

   on P_2 . Find an orthogonal basis of P_2 .

10. Let $w\in R^n$ with $w'w=1$. If $A=I-2ww'$, show that A is symmetric and orthogonal.