淡江大學95學年度碩士班招生考試試題

系別：數學學系　　　　科目：微積分60%及線性代數40%　　23-1

准帶項目請打「V」	
	簡單型計算機

本試題共 2 頁

本試題雙面印製

1. Let $f(x) = \begin{cases} x^2 \sin(1/x), & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$ Find

   (a) (3 points) $\lim_{x \to 0^+} f(x)$

   (b) (3 points) $\lim_{x \to 0^-} f(x)$

   (c) (4 points) Is f continuous? Give reasons to your answer.

2. (a) (5 points) State the Mean Value Theorem(MVT).

   (b) (5 points) Use the MVT to prove the inequality

$$|\sin a - \sin b| \leq |a - b| \text{ for all } a \text{ and } b.$$

3. (a) (5 points) State the Fundamental Theorem of Calculus (FTC).

   (b) (5 points) Find

$$\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1} dt.$$

4. (10 points) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n 4^n}.$$

5. (10 points) Find the absolute maximum and minimum values of

$$f(x, y) = 4xy^2 - x^2y^2 - xy^3$$

   on the set D, the closed triangular region in the xy-plane with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$.

6. (10 points) Show that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

7. (5 points) Let $\mathbf{V}$ be a vector space with basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Prove that $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3\}$ is also a basis for V.

8. Letting P_3 be the vector space of polynomials of degree at most 3 and the ordered basis for P_3 be $B = (x^3, x^2, x, 1)$. Let $T : P_3 \to P_3$ be defined by $T(p(x)) = \frac{d}{dx} p(x)$.

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(a) (5 points) Find the matrix representation A of T.

(b) (5 points) Use A to find $T(4x^3 - 5x^2 + 10x - 13)$.

9. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -8 & 4 & -6 \\ 8 & 1 & 9 \end{bmatrix}$.

(a) (10 points) Find the characteristic polynomial, the real eigenvalues, and the corresponding eigenvectors of A.

(b) (5 points) Find an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$.

10. (10 points) Find an orthonormal basis for the null space of the matrix

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 2 & 5 & 1 & 4 \\ 1 & 1 & 2 & -1 \end{bmatrix}.$$