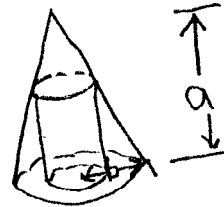准帶項目請打「V」

| | 簡單型計算機 |

本試題共 二 頁

請展示演算過程，否則不予計分，每題 10 分，共 10 題。

1. Prove the following Reduction Formula: for an integer $n \geq 2$,

$$\int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx.$$

Use the formula to evaluate $\int_0^{\pi/2} \sin^8 x \, dx$.

2. Given a right circular cone with disk base of radius b and height a. Find the maximum volume of the inscribed right circular cylinder.

3. Determine whether the series $\sum_{n=1}^{\infty} \dfrac{n^2 2^n}{(2n-1)!}$ is convergent or divergent.

4. Find the maximum and minimum value of

$$f(x,y,z) = x^2 - 2x + y^2 - 4y + z^2 - 4z$$

in the region $x^2 + y^2 + z^2 \leq 36$.

5. Use a triple integral to find the volume of the tetrahedron T enclosed by the coordinate planes and the plane $2x + y + z = 4$.

6. Sketch the curve $y = \dfrac{2x^2}{x^2 - 1}$.

7. Compute the rank of matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix}$; and find bases for the row and column spaces of A.

8. Let $A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

9. Show that the determinant of the Vandermonde matrix

$$\begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}$$

is $(b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$.

背面尚有試題

系別：數學學系　　　　　　　科目：微積分 60% 及線性代數 40%

10. Given the vector space $P_2 = \{a_0 + a_1 x + a_2 x^2 | a_0, a_1, a_2 \in \mathbb{R}\}$ with the inner product $<p, q> = \int_0^1 p(t)q(t)dt$

(1) Starting with $B = \{1, x, x^2\}$, use Gram-Schmidt Orthogonalization to obtain an orthogonal basis $\widetilde{B}$ for P_2

(2) Find the transition matrix P such that, for each $q(x) \in P_2$, $P[q]_B = [q]_Q$, where $B = \{1, x, x^2\}$ and $Q = \{1 + x, 1 - x, x + x^2\}$