淡江大學九十二學年度碩士班招生考試試題

系別：數學學系　　　　科目：微積分 60%及線性代數 40%

准帶項目請打「○」否則打「×」
簡單型計算機

本試題共 1 頁

請展示演算過程，否則不予計分，每題 10 分，共 10 題。

1. Determine whether the series is convergent or divergent.

   (a) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$　(b) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$

2. Find the interval on which the curve

   $y=\int_0^x \frac{1}{1+t+t^2}dt$ is concave upward.

3. Evaluate $\int \frac{2x^2-x+4}{x^3+4x}dx$.

4. (a) Prove that $\int_{-\infty}^{\infty} e^{-x^2}dx=\sqrt{\pi}$. (b) Find $\int_{-\infty}^{\infty} e^{-\frac{x^2}{3}}dx$.

5. Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$, above the xy-plane, and inside the cylinder $x^2+y^2=2x$.

6. Find the volume of the solid that lies above the cone $z=\sqrt{x\ +y^2}$ and below the sphere $x^2+y^2+z^2=z$.

7. Compute the rank of matrix $A=\begin{bmatrix}2 & -1 & 1\\ -2 & 1 & 1\\ 4 & -2 & 3\\ -6 & 3 & 0\end{bmatrix}$, and find bases for the row and column spaces of A.

8. Let A be an n×n matrix with n distinct eigenvalues, prove that A is diagonalizable.

9. Let S be the set of all $(x,y,z)\in R^3$ such that $2x^2+6xy+5y^2-2yz+2z^2+3x-2y-z+14=0$. Find an orthonormal basis β for R^3 such that the equation relating the coordinates of points of S relative to β is simplified. Describe S geometrically.

10. Let $A=\begin{bmatrix}2 & -1 & 0 & 1\\ 0 & 3 & -1 & 0\\ 0 & 1 & 1 & 0\\ 0 & -1 & 0 & 3\end{bmatrix}$, find a Jordan Canonical form of A.