淡江大學九十一學年度碩士班招生考試試題

系別：數學系　　科目：微積分 60%及線性代數 40%

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一. 選擇題: (25%)

1. If $F(x) = \int_e^x \log t\,dt$ for all positive x, then $F'(x) =$

(A) x (B) $\frac{1}{x}$ (C) $\log x$ (D) $x\log x$ (E) $x\log x - 1$

2. Let R be the set of real numbers and let f and g be functions from R to R. The negation of the statement

" For each s in R, there exists an r in R such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

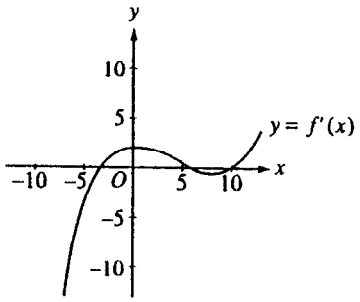
(A) For each s in R, there does not exist an r in R such that if $f(r) > 0$, then $g(s) > 0$.

(B) For each s in R, there exists an r in R such that $f(r) > 0$, and $g(s) \le 0$.

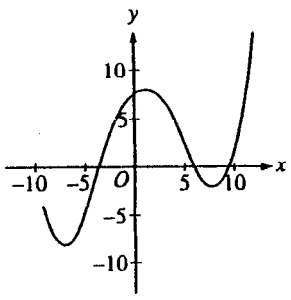
(C) There exists an s in R such that for each r in R, $f(r) > 0$, and $g(s) \le 0$.

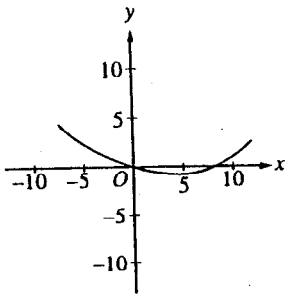
(D) There exists an s in R and there exists an r in R such that $f(r) \le 0$, and $g(s) \le 0$.

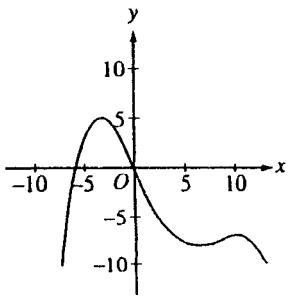
(E) For each r in R, there exists an s in R such that $f(r) \le 0$, and $g(s) \le 0$.

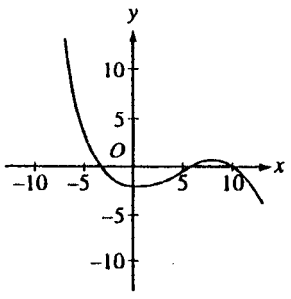


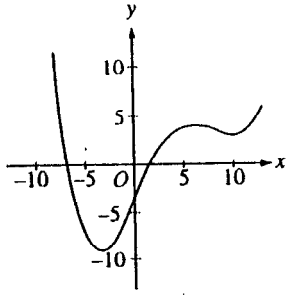
3. If the figure above is the graph of $y = f'(x)$, which of the following could be the graph of $y = f(x)$?

(A)


(B)


(C)


(D)


(E)


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4. If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is

(A) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$　(B) $\frac{1}{a^2+b^2}\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$　(C) $\frac{1}{a^2+b^2}\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

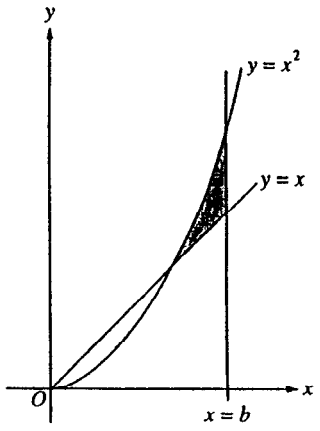
(D) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$　(E) $\frac{1}{a^2-b^2}\begin{pmatrix} -b & a \\ a & b \end{pmatrix}$

5. If A is 3 × 3 matrix such that $A\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $A\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then the product $A\begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}$ is

(A) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$　(B) $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$　(C) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$　(D) $\begin{pmatrix} 9 \\ 10 \\ 11 \end{pmatrix}$　(E) not uniquely determined by the information given.

二. 計算題: (75%)　請務必將計算過程寫出來 否則不予記分!!!

1. (9%)　If　b > 0 and if $\int_0^b x\,dx = \int_0^b x^2\,dx$, then what is the area of the shaded region in the figure below.



2. (6%)　For what value or values of the constant m, if any, is

$$f(x) = \begin{cases} \sin 2x & x \le 0 \\ mx & x > 0 \end{cases}$$

differentiable at x = 0?

3. (10%)　Find the four points on the curve $(x^2 + y^2)^2 = x^2 - y^2$ at which the tangent line is horizontal.

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4. (20%) Evaluate the following integrals.

(1) $\int \frac{x}{\sqrt{x^2+2x-3}}dx$

(2) $\int_0^1 \int_0^x \cos\frac{1}{2}\pi x^2 dydx$

5. (10%) For $0 < t < \pi$, then the matrix $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$ has distinct complex eigenvalues λ_1 and λ_2. Find the value of t, $0 < t < \pi$, such that $\lambda_1 + \lambda_2 = 1$?

6. (10%) Show that the linear transformation $T : R^3 \to R^3$ defined by

$$T([x_1, x_2, x_3]) = [x_1 - 2x_2 + x_3, x_2 - x_3, 2x_2 - 3x_3]$$

is invertible, and find a formula for its inverse.

7. (10%) Find a Jordan Canonical form of the matrix

$$A = \begin{pmatrix} 2 & 5 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$