

淡江大學九十學年度碩士班招生考試試題

系別：數學學系

科目：微積分 60% 及 線性代數 40%

准帶項目請打「○」否則打「×」	
計算機	字典

本試題共 / 頁

請務必將每一題之演算過程寫出來，否則一律不予計分!!!

(10%) 1. Prove that, if $\lim_{x \rightarrow a} f(x)$ exists, then $\lim_{x \rightarrow a} |f(x)| = |\lim_{x \rightarrow a} f(x)|$.

(10%) 2.(a) Use implicit differentiation to compute dy/dx , where

$$x^3 + 6x^2y - 7xy + 3xy^3 + 21 = 0. \quad (1)$$

(b) Find the equation of the line tangent to curve (1) at the point (1,-2).

(10%) 3. Let f be a continuous function such that $f(0) = 0$ and

$$\lim_{x \rightarrow 0} \frac{f(x)}{x} = 2.$$

Define a new function g by $g(x) = \frac{f(x)}{x}$ if $x \neq 0$ and $g(0) = 2$.

(a) Explain(解釋) why g is continuous at all points $a \neq 0$.

(b) Explain why g is continuous at 0.

(10%) 4. Given the following functions.

(a) Find the derivative of $f(x) = \ln[\ln(x^2 + 3x - 5)]$.

(b) Find df of $f(x, y) = \left(\frac{y^2 + 3x - 4}{x^2 + 4y - 3}\right)^4$.

(10%) 5. Evaluate the following integrals.

(1) $\int e^{\sqrt{x}} dx = ?$

(2) $\int xy\sqrt{x^2 + y^2} dx = ?$

(10%) 6. $\int_0^3 \int_{y^2}^9 ye^{-x^2} dx dy$.

(10%) 7. Prove that if V and W are vector spaces of equal(finite) dimension and $T : V \rightarrow W$ is linear, then T is one-to-one if and only if T is onto.

(10%) 8. Let A denote the coefficient matrix of

$$\begin{cases} 2x_2 + 4x_3 = 2 \\ 2x_1 + 4x_2 + 2x_3 = 3 \\ 3x_1 + 3x_2 + x_3 = 1. \end{cases}$$

(a) Show that A is invertible and compute A^{-1} .

(b) Use A to solve the system.

(20%) 9. Find a Jordan canonical form for A from the given data.

(1) Suppose that A is an 8×8 matrix, $A - I$ has nullity 2, $(A - I)^2$ has nullity 4, $(A - I)^k$ has nullity 5 for $k \geq 3$, and $(A + 2I)^j$ has nullity 3 for $j \geq 1$.

(2)

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{pmatrix} = SJS^{-1},$$

where S is a Jordan basis and J is a Jordan canonical form.