淡江大學八十八學年度碩士班招生考試試題

系別：數學學系　　　科目：微積分 60%及線性代數 40%　　　本試題共 1 頁

1. Suppose $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x^2}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0, \end{cases}$ find $f'(0)$ if it exists.

2. Use the definition of the definite integral to evaluate $\int_0^1 x^2\,dx$.

3. (a) $\frac{d}{dx}\int_{x^2}^{x^3}\frac{1}{t}\,dt=?$ (b) $\lim_{(x,y)\to(0,0)}(x^2+y^2)\ln(x^2+y^2)$

4. (a) $\int e^x\cos x\,dx=?$ (b) $\frac{d}{dx}x^x=?$

5. Find power series representation for $\arctan x=\tan^{-1}x$

6 Find the extrema of $f(x,y)=x\cdot y$ if $f(x,y)$ is restricted to the ellipse $4x^2+y^2=4$.

7. Suppose $V_1=(1,-2,0,3)$, $V_2=(2,-5,-3,6)$, $V_3=(0,1,3,0)$, $V_4=(2,-1,4,-7)$, $V_5=(5,-8,1,2)$, and $V=\operatorname{span}(V_1,V_2,V_3,V_4,V_5)$.
 (a) Find a subset of $\{V_1,V_2,V_3,V_4,V_5\}$ that is a basis for V
 (b) Express the vectors not in the basis as a linear combination of the basis vectors.

8. Find bases for the eigenspaces of $A=\begin{bmatrix}0&0&-2\\1&2&1\\1&0&3\end{bmatrix}$.

9. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be defined by $T(x_1,x_2,x_3)=(3x_1+x_2,-2x_1-4x_2+3x_3,5x_1+4x_2-2x_3)$. Determine whether T is one-to-one; if so, find $T^{-1}(x_1,x_2,x_3)$.

10 If $V_1,V_2,\cdots,V_k$ are eigenvectors of a linear transformation A corresponding to distinct eigenvalues $\lambda_1,\lambda_2,\cdots,\lambda_k$. Prove or disprove that $\{V_1,V_2,\cdots,V_k\}$ is linearly independent.

（每題 10 分）