淡江大學八十七學年度碩士班入學考試試題

系別： 數學系　　科目： 微積分 60%及線性代數 40%

Answer all questions . Show all work.

1(7%) Evaluate

$$\lim_{x\to 0}\left(\frac{1^x+3^x+9^x}{3}\right)^{\frac{1}{x}}.$$

2(10%) Find the interval of convergence of

$$\sum_{n=1}^{\infty}\frac{(2x-1)^n}{n\sqrt{n+1}}.$$

3(17%) Evaluate

$$(a)\int\frac{5x+6}{(x-2)(x^2+4)}dx \qquad (b)\int_0^{\infty} sin(x)e^{-3x}dx.$$

4(10%) Find the maximum and minimum values of the given function in the indicated region

$$f(x,y)=x^2-12x+y^2-9y, \qquad x^2+y^2\le 25.$$

5(16%) Evaluate

$$(a)\int_0^2\int_{x^2}^4\frac{1}{1+y^{\frac{3}{2}}}dydx \qquad (b)\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{\frac{3}{2}}dzdydx.$$

6(10%) Find the determinant, adjoint and inverse of $\begin{pmatrix}3&4&3\\5&7&2\\0&0&1\end{pmatrix}$.

7(10%) Let $T:\mathbf{R}^3\to\mathbf{R}^3$ be defined by $T((x,y,z))=(y,0,z)$. Show that T is a linear transformation.

(a) Find a basis for Range T (or Im(T)) and Null space T (or Ker(T)).

(b) What are dim(Range T) and dim(Null space T) ?

8(10%) For the following matrix and bases find the represented linear transformation, $A=\begin{pmatrix}3&-3&1\\-1&1&0\end{pmatrix}$, $\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$ and $\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}-1\\-2\end{pmatrix}\right\}$.

9(10%) Find the eigenvalues and associated eigenvectors of $\begin{pmatrix}3&1&0\\-4&-1&0\\4&-8&-2\end{pmatrix}$