16-1

淡江大學 104 學年度碩士班招生考試試題

系別：數學學系 B 組　　科目：機率與統計

考試日期：3 月 8 日（星期日）第 3 節　　本試題共 7 大題，2 頁

本試題雙面印刷

1. (10%) An urn contains 25 red balls, 12 black balls and 18 white balls. Randomly draw 10 balls one-at-a-time. Let X be the number of red balls and Y be the number of black balls of the 10-drawing balls. Find the joint p.m.f. of (X, Y)

a) if the drawing is without replacement;

b) if the drawing is with replacement.

2. (20%) Let X and Y have the joint p.d.f. $f(x, y) = 2, \ 0 \le y \le x \le 1$

a) Find $P(0 \le X \le \frac{1}{4}, \ 0 \le Y \le \frac{1}{4})$.

b) Find the conditional mean $E(Y|X = x)$.

c) Find the covariance of X and Y.

d) Are X and Y independent? Why?

3. (15%) Let X_1, X_2 be i.i.d. random variables from $N(0, \sigma^2)$. Find the p.d.f. of random variable $Y_1 = \frac{X_1}{X_2}$.

4. (15%) Let X be a binomial random variable, $B(n, \theta)$. Suppose that we want to estimate the variance of X, Var$X$. Consider the estimator $T(X) = n \cdot \frac{X}{n} \cdot (1 - \frac{X}{n})$. Is $T(X)$ unbiased?

5. (15%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the p.m.f.:

$$f(x; \theta) = \theta(1 - \theta)^{x-1}, \ \ x = 1, 2, 3, \ldots \ \text{ where } \ 0 < \theta < 1.$$

a) Find a sufficient statistic for θ.

b) Find the maximum likelihood estimator, $\hat{\theta}$, of θ.

c) Is $\hat{\theta}$ unbiased?

6. (10%) Let X be a random variable with the p.d.f. $f(x; \theta) = \frac{1}{\theta}, \ 0 < x < \theta$. Consider the hypotheses $H_0 : \theta = \frac{4}{3}$ against the alternative hypotheses $H_1 : \theta = \frac{7}{3}$. Using a single observation of X and the critical region $\{X > 1\}$.

a) Find the significance level of this test;

b) Find the probability of the type II error.

背面尚有試題

16-2

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7. (15=5+10%) Let $X_1, X_2, \ldots, X_n$ be independent random variables distributed as $N(\theta, 1)$.

a) Find $E(\bar{X}^2)$.

b) Find the **UMVUE** of $g(\theta) = \theta^2$.