

# 淡江大學 101 學年度碩士班招生考試試題

系別：數學學系

科目：機 率 論

考試日期：2 月 26 日(星期日) 第 4 節

本試題共 6 大題，1 頁

1. A pair of dice is thrown twice.

(a) (10%) What is the probability of getting totals of 7 and 11?

(b) (10%) What is the probability of getting a total of 7 or 11?

2. (10%) If events  $A, B$ , and  $C$  are mutually independent, prove that events  $A$  and  $(B \cup C)$  are also independent.

3. (15%) Let random variables  $X_1$  and  $X_2$  have joint probability density function (pdf) given by

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & 0 \leq x_1; 0 \leq x_2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the density function for  $Y = X_1 + X_2$ .

4. Let random variables  $X$  and  $Y$  have joint pdf given by

$$f(x, y) = 1 \text{ for } 0 < x < 1, x < y < x + 1.$$

(a) (10%) Find the marginal pdf of  $X$ , and its mean ( $\mu_X$ ) and variance ( $\sigma_X^2$ ).

(b) (10%) Find the marginal pdf of  $Y$ , and its mean ( $\mu_Y$ ) and variance ( $\sigma_Y^2$ ).

(c) (5%) Find the expected value of  $XY$ , i.e.,  $E(XY)$ .

(d) (5%) Find the correlation ( $\rho_{XY}$ ) of  $X$  and  $Y$ .

5. (15%) Let random variables  $X, Y$  and  $Z$  have joint pdf given by

$$f(x, y, z) = \frac{xyz}{108} \text{ for } x = 1, 2, 3; y = 1, 2, 3; z = 1, 2$$

Find the conditional expectation of the random variable  $Z^2$  given  $X = 1$  and  $Y = 2$ .

6. (10%) A random variable  $X$  has a mean  $\mu = 8$  and a variance  $\sigma^2 = 9$ . Estimate  $P(|X - 8| \geq 6)$ .