

# 淡江大學 100 學年度碩士班招生考試試題

19

系別：數學學系

科目：機 率 論

考試日期：2月28日(星期一) 第4節

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1. (10%) Suppose that  $A, B,$  and  $D$  are events such that  $A$  and  $B$  are independent,

$$P(A \cap B \cap D) = 0.04, P(D|A \cap B) = 0.25, \text{ and } P(B) = 4P(A).$$

Determine  $P(A \cup B)$ .

2. (10%) Let  $X$  and  $Y$  be independent random variables. Each has possible values 0, 1, and 2, with probabilities 0.2, 0.3, and 0.5, respectively.

- a) (5%) Find the value of  $P(|X - Y| = 1)$ .  
 b) (5%) Find  $P(X = x|Y = 1)$ .

3. (20%) The sales of a convenience store on a randomly selected day are  $X$  thousand dollars, where  $X$  is a random variable with a distribution function of the following form:

$$F(t) = \begin{cases} 0 & t < 0, \\ t^2/2 & 0 \leq t < 1, \\ k(4t - t^2) & 1 \leq t < 2, \\ 1 & t \geq 2. \end{cases}$$

Suppose that this convenience store's total sales on any given day are less than 2000 dollars.

- a) (5%) Find the value of  $k$ .  
 b) (15%) Let  $A$  and  $B$  be the events that tomorrow the store's total sales are between 500 and 1500 dollars, and over 1000 dollars, respectively. Are  $A$  and  $B$  independent events?

4. (15%) Suppose that three random variables  $X_1, X_2,$  and  $X_3$  have a continuous joint distribution with the following joint probability density function

$$f(x_1, x_2, x_3) = \begin{cases} ce^{-(x_1+2x_2+3x_3)} & \text{for } x_i > 0 \ (i = 1, 2, 3), \\ 0 & \text{otherwise.} \end{cases}$$

Determine

- a) (5%) the value of the constant  $c$ ;  
 b) (10%)  $P(X_1 < 1|X_2 = 2, X_3 = 1)$ .

5. (15%) Suppose that  $X$  is a random variable for which the moment generating function is

$$\phi(t) = \frac{e^t}{5} + \frac{2e^{4t}}{5} + \frac{2e^{8t}}{5} \text{ for } -\infty < t < \infty.$$

- a) (5%) What is the probability mass function of  $X$ ?  
 b) (10%) Compute  $Var(X)$ .

6. (15%) Suppose that the joint probability density function of two random variables  $X$  and  $Y$  is

$$f(x, y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2} \text{ for } -\infty < x < \infty \text{ and } -\infty < y < \infty.$$

Find  $P(-\sqrt{2} < X + Y < 2\sqrt{2})$  in terms of  $\Phi(z)$ , where  $\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} dw$ .

7. (15%) Let  $X_1, X_2, \dots, X_n$  be independent random variables, each having the uniform distribution on  $(0, 1)$ . Let  $Y = -2 \ln(X_1 X_2 \cdots X_n)$ . What is the distribution of  $Y$ ?