1.(5%) Suppose that A and B are independent events such that the probability that neither occurs (二個事件均不發生的機率) is a , and P(B) = b , find P(A).

2.(20%) A random variable X has the following cdf (cumulative distribution function) :

$$F(x)=\begin{cases}0 & x<1\\.30 & 1\leq x<3\\.40 & 3\leq x<4\\.45 & 4\leq x<6\\.60 & 6\leq x<12\\1 & 12\leq x\end{cases}$$

Find (a) $P(4\leq X\leq 6)$

(b) $P(X\geq 6)$

3. Let Y be a binomial r.v.(random variable) with parameters n and p,
(a) (5%)What is the p.f.(probability function, 機率函數,也稱 p.m.f.) of Y ?
(b) (10%) Using the probability function, derive(導出) $E[X(X-1)]$.

4. (10%) R.v. Y has probability density function $f(y) = (3/2) y^2$, $-1\leq y\leq 1$, find the c.d.f. of U $= 2-Y$.

5. (20%) X,Y are r.v.'s with joint pdf $f(x,y) = 24xy$, $0\leq x\leq 1, 0\leq y\leq 1$, $x+y\leq 1$，Find
(a) the joint pdf of X+Y and X.
(b) p.d.f. of X+Y.

6. (20%) X~ Poisson(μ)
(a) 導出 X 之 mgf(moment generating function)。
(b) 若 X_1,X_2 為 independent Poisson r.v.'s, 參數分別為 μ_1, μ_2，利用 mgf 來找 X_1+X_2 之分布。

7. (10%) Let X_1, X_2, …,X_n be a random sample from a uniform(0,1) distribution, and let $\overline{X}$ be the sample mean. Use the Central Limit Theorem to find $P(0.45 < \overline{X} < 0.55)$ in terms of the cdf $\Phi(x)$ (答案用 N(0,1)之 cdf $\Phi(x)$表示)