淡江大學 97 學年度碩士班招生考試試題

系別：數學學系　　　　　　　　科目：機　率　論

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1. **(25%)** A common test for AIDS is called the ELISA test (Enzyme-Linked Immunosorbent Assay). Among 1,000,000 people who are given the ELISA test, we can expect results similar to those given in the table.

	B_1: Carry AIDS Virus	B_2: Do Not Carry AIDS Virus	Totals
A_1: Test Positive	4,885	73,630	78,515
A_2: Test Negative	115	921,370	921,485
Totals	5,000	995,000	1,000,000

If one of these 1,000,000 people is selected randomly, find the following probabilities: (a) $P(B_1)$, (b) $P(A_1)$, (c) $P(A_1|B_2)$, (d) $P(B_1|A_1)$, (e) In words, what do parts (c) and (d) say?

2. **(15%)** Let the random variable X have the p.m.f. $f(x)=\frac{(|x|+1)^2}{9}$, $x=-1,0,1$. Compute $E(X), E(X^2)$, and $E(3X^2-2X+4)$.

3. **(20%)** The random variable X has a gamma distribution, denoted by $X \sim \Gamma(\alpha,\theta)$, if its p.d.f. is defined by $f(x)=\frac{1}{\Gamma(\alpha)\theta^{\alpha}}x^{\alpha-1}e^{-x/\theta}, 0\le x<\infty$.
   (a) Find the moment-generating function of X.
   (b) Show that $E(X)=\alpha\theta$ and $Var(\theta)=\alpha\theta^2$.

4. **(10%)** Let X_1 and X_2 be two independent random variables with respective means μ_1 and μ_2 and variances σ_1^2 and σ_2^2. Show that the mean and the variance of $Y=X_1X_2$ are $\mu_1\mu_2$ and $\sigma_1^2\sigma_2^2+\mu_1^2\sigma_2^2+\mu_2^2\sigma_1^2$, respectively.

5. **(20%)** Let the independent random variables X_1 and X_2 be $N(0,1)$ and $\chi^2(\gamma)$, respectively. Let $Y=X_1/\sqrt{X_2/\gamma}$ and $Z=\sqrt{X_2/\gamma}$.
   (a) Find the joint p.d.f. of Y and Z. (Hint: $\chi^2(\gamma)\equiv\Gamma(\gamma/2,2)$)
   (b) Determine the marginal p.d.f. of Y.

6. **(10%)** Let X equal the birth weight in grams of a baby born in the Taiwan. Assume that E(X) = 3320 and Var(X) = 660^2. Let $\bar{X}$ sample mean of a random sample of size n = 225. Find $P(3233.76\le\bar{X}\le 3406.24)$, approximately.