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淡江大學96學年度碩士班招生考試試題

系別：數學學系　　科目：機　率　論

准帶項目請打「V」	
	簡單型計算機

本試題共 / 頁

1. (10%) If the events A_j, $j = 1, 2, 3$ are such that $A_1 \subset A_2 \subset A_3$ and $P(A_1) = \frac{1}{4}$, $P(A_2) = \frac{1}{3}$, $P(A_3) = \frac{7}{12}$.

a) Calculate $P(A_1 \cap A_2^c \cap A_3^c)$

b) Calculate $P(A_1^c \cap A_2 | A_3)$

2. (10%) Suppose that X_1 and X_2 are i.i.d. continuous $Uniform(1, b)$. Let $Y = X_1 X_2$. Find the probability density function of Y.

3. (20%) Roll a red and a black die (six-sided). Let X be the outcome on the red die and Y be the sum of the two dice.

a) Find the joint probability density function of X and Y.

b) Calculate $E(Y|X = x)$.

4. (20%) Suppose that random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} p^2(1-p)^y & x, y = 0, 1, 2, \ldots \text{ and } x \le y \\ 0 & \text{otherwise} \end{cases}$$

where $0 < p < 1$.

a) Find the marginal probability density functions of X and Y.

b) Calculate $Cov(X, Y)$. Are X and Y independent?

5. (20%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with the probability density function $f(x)$ and the cumulative distribution $F(x)$. We say a record occurs at time n and set $Y_n = 1$ if $X_n > \max\{X_1, X_2, \ldots, X_{n-1}\}$, otherwise $Y_n = 0$. Let $R_n = Y_1 + Y_2 + \ldots + Y_n$ be the number of records that have been set by time n.

a) Find $P(Y_n = 1)$.

b) Find $E(R_n)$.

6. (20%) Let $X_1, X_2, \ldots, X_n$ be i.i.d. continuous $Uniform(0, 1)$. Let $M_n = \min\{X_1, X_2, \ldots, X_n\}$.

a) Show that $M_n \to 0$ in probability.

b) Find $\lim_{n\to\infty} P\left(M_n > \frac{x}{n}\right)$.