淡江大學 95 學年度碩士班招生考試試題

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系別：數學學系　　　　　　　　科目：機　率　論

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

1.(10%) Customers arrive in a certain shop according to an approximate Poisson process at a mean rate of 20 per hour. Let X denote the waiting time in minutes until the first customer arrives.

i). Find the cdf $F(t) = P(X \le t)$ of X.

ii). Show that $\lambda(t) = \frac{F'(t)}{S(t)}$ is a constant where $S(t) = 1 - F(t)$.

2.(10%) Let X and Y be two independent Poisson random variables with mean λ_1 and λ_2. Find the conditional probability $P(X = x \mid X + Y = k)$, where $x = 0, 1, \cdots, k$.

3.(15%) At a hospital's emergency room, patients are classified and 10% of them are critical, 30% are serious, and 60% are stable. Of the critical ones, 40% die; of the serious, 20% die; and of the stable, 10% die. Given that a patient dies, what is the conditional probability that the patient was classified as critical.

4.(15%) Let X equal the number of flips of a fair coin that are required to observe one head.

i). Show that $P(X > k + j \mid X > k) = P(X > j)$ for nonnegative integers k and j.

ii). Show that the moment generation function of X is $\frac{pe^t}{1-(1-p)e^t}$ where $p = 1/2$.

iii). Compute $E(X)$ and $Var(X)$.

5.(10%) A drunken men has n keys, one of which open the door to his office. He tries the keys at random. Compute the mean and the variance of the number of trials to open the door if the wrong keys (a) are eliminated; (b) are not eliminated. (Hint: The mgf in problem 4 may be helpful !)

6.(20%) Let X_1 and X_2 have independent gamma distribution with parameters α, θ and β, θ. That is, the joint p.d.f. of X_1 and X_2 is

$$f_(x_1, x_2) = \frac{1}{\Gamma(\alpha)\Gamma(\beta)\theta^{\alpha+\beta}} x_1^{\alpha-1} x_2^{\beta-1} exp(-\frac{x_1 + x_2}{\theta}).$$

Find the distribution of $Y = \frac{X_1}{X_1+X_2}$.

7.(20%) Let $X_1, \cdots, X_n$ be i.i.d. random variables with mean $E(X_1) = \mu$ and variance $Var(X_1) = \sigma^2$.

i). Prove that for any $\epsilon > 0$, one has $P(\mid X_1 - \mu \mid \ge \epsilon) \le \frac{\sigma^2}{\epsilon^2}$.

ii). Prove the weak law of large numbers $P(\mid \frac{\sum_1^n X_i}{n} - \mu \mid \ge \epsilon) \to 0$ for any $\epsilon > 0$.