淡江大學九十四學年度碩士班招生考試試題 31-1

系別：數學學系　　　科目：機　率　論

准帶項目請打「V」	
	簡單型計算機

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本試題雙面印製

1. A sequence of independent *Bernoulli* trials, each with probability of success p. Let X denote the number of trials of failures before the first success. That is, X is a *geometric* random variable with probability function

$$P\{X=k\}=(1-p)^k p, k=0,1,\ldots$$

(a) Show that $EX=(1-p)/p$. (10%)

(b) Let Y denote the number of trials of failures before the outcome of the *r-th* success. Write down the probability function $P\{Y=k\}$ of Y. (5%)

(c) Write $Y=Y_1+\ldots+Y_r$, where Y_i is the number of trials of failures to go from a total of $i-1$ to a total of i successes. Show that $EY=r(1-p)/p$. (5%)

(d) The moment generating function (m.g.f.) of Y is defined as Ee^{tY}. Show that the m.g.f. of the random variable Y in (c) is given by

$$\frac{p^r}{\left[1-(1-p)e^t\right]^r}, t<-\log(1-p). \text{ (10\%)}$$

(Hint: $\sum_{x=0}^{\infty}\binom{r+x-1}{x}a^x=(1-a)^{-r}, |a|<1$.)

(e) Use the result in (d) to show that the mean and variance of the random variable Y in (c) is given by

$$EY=r(1-p)/p,\quad Var(Y)=r(1-p)/p^2. \text{ (10\%)}$$

2. Let (X,Y) have joint density function $f(x,y)=8xy, 0<x<y<1$.

(a) Show that the conditional density function of X given $Y=y$ is

$$f(x\mid y)=2x/y^2, 0<x<y<1. \text{ (10\%)}$$

(Hint: What is the marginal density function of Y?)

(b) Show that the conditional variance of X given $Y=y$ is

◀注意背面尚有試題▶

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(c) Calculate the variance of X by completing the following equation

$$Var(X) = E[Var(X \mid Y)] + Var[E(X \mid Y)].\ (10\%)$$

3. Let X be a continuous random variable with density function $f_X(x) = e^{-x}, x > 0$. Let $Y = X^{1/2}$.

   (a) Find the density function of Y. (10%)

   (b) Let $U = F(Y)$, where $F(Y)$ is the distribution function of Y. Show that U is uniformly distributed on $[0,\ 1]$. (10%)

   (Hint: $P\{U \le u\} = P\{F(Y) \le u\}$.)

   (c) Suppose now that you want to simulate a random variable Y from the computer. Use the property in (b) to find the transformation function $Y = G(U)$, so that Y may be obtained from the computer generated random variable U after the $G(.)$ function mapping, where U is $Uniform[0,\ 1]$. (10%)

   (Hint: What is the inverse function of $F(Y)$?)