淡江大學九十三學年度碩士班招生考試試題

系別：數學學系　　科目：機　率　論

准帶項目請打「○」否則打「×」
簡單型計算機　×

本試題共 / 頁

1. (20%) Two digits are chosen at random without replacement from the set of integers $\{1,2,3,4,5,6,7,8\}$.

a) Find the probability that both digits are greater than 5.

b) Show that the probability that the sum of the digits will be equal to 5 is the same of the probability that their sum will exceed 13.

2. (20%) If the moment-generating functions of X_1 and X_2 are $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^4$ and $\frac{2}{5}e^t+\frac{1}{5}e^{2t}+\frac{2}{5}e^{3t}$, respectively. Suppose that X_1 and X_2 are independent. Find

a) $P(X_1=3)$

b) $P(X_1+X_2=4)$

c) $Var(X_1)$

d) the moment-generating function of $Y=2X_1-3X_2$.

3. (20%) The joint probability mass function of X and Y is

$$f(x,y)=\frac{1}{18},\quad 0\le x\le 5,\quad x\le y\le x+2,$$

where x and y are nonnegative integers.

a) Find the marginal probability mass function of Y.

b) Find $E(Y|x)$.

4. (20%) Let X_1 and X_2 be a random sample of size 2 from a distribution with probability density function $f(x)=\frac{1}{3}e^{-\frac{x}{3}}$, $x>0$. Consider $Y_1=X_2-X_1$ and $Y_2=X_1+X_2$. Find

a) the marginal probability density function of Y_1;

b) the conditional probability density function of $Y_1|y_2$.

5. (20%) Let $X_1, X_2, \ldots$ be a sequence of discrete random variables such that X_n has the probability mass function f_n given by $f_n(0)=1-\frac{1}{n}$ and $f_n(n)=\frac{1}{n}$.

a) Find $E(X_n)$ and $Var(X_n)$.

b) Show that $X_n \xrightarrow{P} 0$, i.e., the sequence $\{X_n\}$ converges to 0 in probability.