

淡江大學九十二學年度碩士班招生考試試題

系別：數學學系

科目：機 率 論

准帶項目請打「○」否則打「×」
簡單型計算機 X

本試題共 1 頁

1. (10%) Suppose that in the families with two children, all sex distribution are equally probable.
 - a) If a family is selected at random and is found to have a girl. What is the probability that the other child of this family is a girl.
 - b) If a child is selected at random and is found to be a girl. What is the probability that the other child of this girl's family is also a girl?

2. (15%) Urn I contains 5 white and 20 black balls. Urn II contains 15 white and 10 black balls. An urn is selected at random and one of its balls is drawn randomly and observed to be black and then return to the urn. What is the probability that the second ball is black if it is drawn from the same urn.

3. (20%) An absent-minded professor wrote n letters and sealed them in envelopes before writing the addresses on the envelopes. So he wrote the n addresses on the envelopes at random. Let X denote the number of letters that were addressed correctly. Calculate the expectation and variance of X .

4. (5, 10, 10%) At an intersection of two roads, the vehicles arriving are either cars or trucks. Suppose that the cars arrive at the intersection at a Poisson rate of λ per minute, and trucks arrives at a Poisson rate of μ per minute.
 - a) Let X be the time until the next arrival of car, and Y be the time for the truck. Show that X is exponential distributed with mean $1/\lambda$.
 - b) What is the probability of the event "The next vehicle arriving is a car"?
 - c) When condition on the event in b), what is the expectation of the time until the next vehicle arrives?

5. (20%) Suppose the distribution of the random vector (X_1, X_2, \dots, X_r) is multinomial with parameter $(n, p_1, p_2, \dots, p_r)$, where $\sum_1^r p_i = 1$. Find the moment generating function $M(t_1, t_2, \dots, t_r)$ and use it to calculate the correlation coefficient between X_1 and X_r .

6. (10%) Suppose the distribution of X_i is Binomial(i, p_i) and Y has distribution Poisson(λ). Assume that ip_i converge to λ as $i \rightarrow \infty$, show that the sequence $\{X_i\}$ converge to Y in distribution.