淡江大學九十學年度碩士班招生考試試題

系別：數學學系

科目：機　率　論

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本試題共 / 頁

1. (20%) Consider two urns U_i, $i = 1, 2$ such that urn U_1 contains 10 white balls and 5 black balls, and urn U_2 contains 6 white balls and 9 black balls.

a) A ball is drawn at random from each one of the two urns and is placed into a third urn. Then a ball is drawn at random from the third urn. Find the probability that the ball is black.

b) A balanced die is rolled and if an even number appears a ball, chosen at random from urn U_1, is transferred to urn U_2. If an odd number appears, a ball, chosen at random from urn U_2 is transferred to urn U_1. Find the probability that, after the above experiment is performed twice, the number of white balls in urn U_2 remains the same.

2. (15%) Let X be a continuous random variable with p.d.f. $f(x) = (\beta - \alpha)e^{-(\beta-\alpha)x}$, $x > 0$. If $Y|X = x$ has a Poisson distribution with parameter αx. Find the p.d.f. of Y.

3. (20%) Let the random variables X, Y be jointly distributed with p.d.f.

$$f(x, y) = \frac{2}{n(n+1)}, \quad x = 1, 2, \ldots y; \ y = 1, 2, \ldots n.$$

a) Find $E(X|Y = y)$.

b) Find $E(Y|X = x)$.

4. (15%) Let (X, Y) be jointly uniformly distributed on the triangle $0 < x < y < 1$. Let $U = X + Y$. Find the p.d.f. of U.

5. (15%) Let X_1, X_2, X_3 be i.i.d. random variables with p.d.f. $f(x) = e^{-x}$, $x > 0$. Let $U_1 = X_1 + X_2 + X_3$, $U_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}$, and $U_3 = \frac{X_1}{X_1 + X_2 + X_3}$. Find the jointly p.d.f. of (U_1, U_2, U_3).

6. (15%) Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables with $E(X_n) = \mu_n$ and $Var(X_n) = \sigma_n^2$. If $\mu_n \to a$ and $\sigma_n^2 \to 0$ as $n \to \infty$. Show that $X_n \xrightarrow{P} a$.