

## 淡江大學八十九學年度碩士班招生考試試題

系別：數學學系

科目：機率論

本試題共 / 頁

一. Assume that  $(X, Y)$  is a discrete random vector with joint frequency function  $f(x, y) = \frac{1}{4}(x+y)$ ,  $(x, y) = (0, 1), (1, 0), (1, 1)$ .

(10%)

Find  $E(Y|X=1)$ .

二. Let  $(X, Y)$  be a continuous random vector with joint density function  $f(x, y) = 8xy$ ,  $0 < x < y < 1$ .

(25%)

(a) Find the conditional density of  $X$  given  $Y=y$ .(b) Evaluate  $P(X+Y < 1)$ .

三. Suppose  $X$  has density  $f(x) = \frac{1}{4}$ ,  $-1 \leq x \leq 3$ , and let  $U = X^2$ .

(20%)

(a) Find the distribution function of  $U$ .(b) Find the density function of  $U$ .

四. Let  $X$  and  $Y$  be independently uniformly distributed on  $(0, 1)$ .

(15%)

Let  $W = \frac{X}{Y}$ . Find the density function of  $W$ .

五. Let  $A, B$  and  $C$  are independent events. Show that  $A^c, B^c$  and  $C^c$  are also independent.

(15%)

六. Let  $X_n \sim B(n, \frac{a}{n})$ . Show that  $X_n \xrightarrow{d} X \sim \text{Poisson}(a)$ .

(15%)