淡江大學八十八學年度碩士班招生考試試題

系別：數學學系　　科目：機　率　論　　本試題共 1 頁

(一). Let $X_1, \cdots, X_n$ be independent, identically distributed with finite
15% mean μ. Then show that $\overline{X}_n$ converges to μ in probabity as $n \to \infty$.

(二) Let $X_n \xrightarrow{d} X$. Then show that $P(X_n < a) \longrightarrow P(X < a)$ as $n \to \infty$
15% for all a where F is continuous.

(三) A point (a, b) is selected at random from the square $-1 \le a, b \le 1$.
15% Find the probabity that the equation $ax^2 + bx + c = 0$ has two distinct real solutions.

(四) An animal lays(生) a certain number X of eggs, where X
15% is random and has the Poisson distribution, $Po(\lambda)$. Each egg hatches(孵) with probabity p independent of hatching of other eggs. Determine the distribution of Y = the number of eggs that hatch.

(五) Assume that X and Y have joint density
20% $$f(x, y) = 4xy \qquad 0 \le x, y \le 1$$
Determine (i) the conditional density of X given $X - Y = 0$.
(ii) the conditional density of X given $Y/X = 1$.

(六) Let X have density $f(x) = 2(1-x)$ for $0 \le x \le 1$.
20% Find the density of (i) $Y = X(1-X)$, (ii) $Z = \max(X, 1-X)$.