淡江大學八十七學年度碩士班入學考試試題

系別： 數學系　　　　科目： 機率論

1. (10%) Let X_1 and X_2 be a random sample of size $n = 2$ from a distribution with probability density function $f(x) = 6x(1-x)$, $0 < x < 1$. Find the mean and the variance of $Y = X_1 + X_2$.

2. (15%) If the joint probability density function of X and Y is given by

$$f(x, y) = 6e^{-3x-2y}, \qquad x > 0, y > 0.$$

Find the probability density function of the random variable $W = X + Y$.

3. (15%) Given the joint probability density function of X and Y

$$f(x, y) = \frac{2}{3}(x + 2y), \qquad 0 < x < 1, \ 0 < y < 1.$$

Find the conditional mean and the conditional variance of X given $Y = 1/2$.

4. (15%) If $X \sim N(\mu, \sigma^2)$, let $W = e^X$. Find the median and the mode of W.

5. (15%) Given the independent random variables X_1, X_2 and X_3 with the probability density functions

$$f_1(x_1) = e^{-x_1}, \ f_2(x_2) = 2e^{-2x_2}, \text{ and } f_3(x_3) = 3e^{-3x_3}.$$

where $x_1 > 0$, $x_2 > 0$, and $x_3 > 0$. Evaluate the probability

$$P(X_1 + X_2 \le 1, \ X_3 > 1).$$

6. (15%) A consulting firm rent cars from three agencies: 60% from agency 1, 30% from agency 2, and 10% from agency 3. If 9% of the cars from agency 1 need a tune-up, 20% of the cars from agency 2 need a tune-up, and 6% of the cars from agency 3 need a tune-up,
(a) what is the probability that a rental car delivered to the firm will need a tune-up?
(b) if a car delivered to the firm needs a tune-up, what is the probability that it came from rental agency 2?

7. (15%) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends on the positive integer n. Let U_n converge in probability to the positive constant c and let $P_r(U_n < 0) = 0$ for every n. Prove that the random variable $\sqrt{U_n}$ converges in probability to $\sqrt{c}$.