淡江大學 100 學年度碩士班招生考試試題

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系別： 數學學系　　　　科目：統　計　學

考試日期：2 月 28 日(星期一) 第 3 節　　　　本試題共 7 大題， 1 頁

1.(20%)Let $X_1, X_2, \cdots, X_5$ be a random sample from the $N(\mu, \sigma^2)$ distribution, consider the following 4 statistics: $T_1 = 2X_1 - X_2$, $T_2 = \dfrac{X_1 + X_2 + X_3 + X_4 - X_5}{3}$, $T_3 = \dfrac{X_5 - X_1}{2}$,

$T_4 = \dfrac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$,

(a) Which of the 4 statistics are unbiased estimators of μ ?

(b) Find the variances of all the unbiased estimators of μ, which one has the smallest variance?

2. (20%) Let X be a random variable with probability function under H_0 and H_1 given by

x	1	2	3	4	5	6
$f_0(x)$	0.01	0.01	0.01	0.01	0.01	0.95
$f_1(x)$	0.05	0.04	0.03	0.02	0.01	0.85

(a) Let $\alpha = 0.03$, use the Neymen-Pearson Lemma to find the most powerful test.

(b) Find the power of the most powerful test found in (a).

3.(10%) $X_1, X_2, \cdots, X_k$ is a random sample from the binomial(n, θ) distribution, find a MLE for θ^2.

4. (10%)Let X_1, X_2 be i.i.d random variables with the following p.d.f. $f(\cdot;\theta), \theta > 0$: $f(0;\theta) = e^{-\theta}$, $f(1;\theta) = \theta e^{-\theta}$, $f(2;\theta) = 1 - e^{-\theta} - \theta e^{-\theta}$, and $f(x;\theta) = 0$ otherwise. Use the definition of sufficiency to show that $X_1 + X_2$ is not a sufficient statistic for θ.

5. (20%)Let $\overline{X}$ and $\overline{Y}$ be the sample means of independent random samples of sizes n_1 and n_2 taken from normal populations with known variances σ_1^2, σ_2^2 respectively.

(a) What is a 100($1-\alpha$)% confidence interval for $\mu_1 - \mu_2$?（所用符號都要定義清楚）

(b) If σ_1^2, σ_2^2 are unknown but equal, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, find an appropriate estimator for σ^2.

6. (10%) Let $X_1, X_2, \cdots, X_n$ be a random sample with p.d.f. given by

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0, \quad 0 < \theta < \infty$$

Derive the UMP(uniformly most powerful) test for testing $H_0: \theta \geq \theta_0$ against $H_1: \theta < \theta_0$ at level α.

7. (10%) $X_1, X_2, \cdots, X_n$ is a random sample from $N(\mu, \sigma^2)$ distribution, μ, σ^2 unknown. Find the UMVUE for $\dfrac{\mu}{\sigma}$.