淡江大學99學年度碩士班招生考試試題

21-1

系別：數學學系　　科目：統　計　學

准帶項目請打「V」	
	計 算 機

本試題共 2 頁，5 大題

本試題雙面印製

1. (20%)An observation Z takes one of four values according to one of the three distributions shown in the following table of probabilities:

	θ_0	θ_1	θ_2
z_1	0.2	0.5	0.3
z_2	0.3	0.1	0
z_3	0.1	0.2	0.4
z_4	0.4	0.2	0.3

Consider testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$ or θ_2

(a) Find the likelihood ratio statistic Λ.

(b) Determine all critical regions for Z defined by rules of the form $\Lambda < K$

(c) Find size α for each test in (b).

(d) Compare the two critical regions $\{z_2\}$ and $\{z_1, z_3\}$ in terms of type I and type II errors when $\theta=\theta_2$. Which is the better test?

2. (20%)Let $X_1, X_2, ..., X_n$ be random sample from the distribution of X, which has pdf $f(x)$. Let $T=T(X_1, X_2, ..., X_n)=med\{X_1, X_2, ..., X_n\}$ be the sample median.

(a) Show that $T(X_1+b, X_2+b, ..., X_n+b)=T(X_1, X_2, ..., X_n)+b$

$T(-X_1, -X_2, ..., -X_n)=-T(X_1, X_2, ..., X_n)$

(b) Suppose $\mu=E(X)$ exists and, further, that the pdf $f(x)$ is symmetric about μ,

(1)Show that $(X_1-\mu, X_2-\mu, ..., X_n-\mu)$ and $(-(X_1-\mu), -(X_2-\mu), ..., -(X_n-\mu))$ have the same distribution.

(2) Show that T is an unbiased estimator of μ.

3. (20%)Let $X_1, X_2, ..., X_n, X_{n+1}$ be random sample of size n+1, $n>1$, fro, a distribution that is $N(\mu, \sigma^2)$. Let $\bar{X}=\sum_{i=1}^{n} X_i/n$ and

$S^2=\sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$

(a) Find the constant c so that the statistic $c(\bar{X}-X_{n+1})/S$ has a t-distribution.

(b) If n=8, determine k such that the observed interval $(\bar{x}-ks, \bar{x}+ks)$ is an 80 percent prediction interval for X_9, that is to determine k such that

$P(\bar{X}-kS<X_9<\bar{X}+kS)=0.8$

4. (20%)Let $X_1, X_2, ..., X_n$ be random sample of size $n>2$ from a distribution with pdf $f(x;\theta)=\begin{cases}\theta x^{\theta-1} & \text{for } 0<x<1\\ 0 & eleswhere\end{cases}$ where the parameter space is $\Omega=(0,\infty)$.

(1) What is the mle $\hat{\theta}$ of θ.

(2) Find the variance of the $\hat{\theta}$.

(3) Show that $2\theta n/\hat{\theta}$ has a $\chi^2(2n)$

(4) Use part (3) to obtain a $(1-\alpha)100\%$ confidence interval for θ

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5. (20%)Let X have the pdf $f(x;\theta)=\begin{cases}1/2\theta & \text{for } -\theta<x<\theta \\ 0 & \textit{eleswhere}\end{cases}$ where $\theta>0$

(a) Is the statistic $Y=|X|$ a sufficient statistic for θ? Why

(b) Let $f_Y(y;\theta)$ be the pdf of Y. Is the family $\{f_Y(y;\theta):\theta>0\}$ complete? Why?