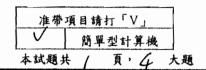
淡江大學 98 學年度碩士班招生考試試題

7

系別:數學學系

科目:統 計 學



1. (20%) Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with p.d.f. $f(x; p) = p^x (1-p)^{1-x}, x = 0, 1$.

Then

- (a) (10%) Show that $Y = X_1 + X_2 + \cdots + X_n$ is a sufficient statistics for p.
- (b) (10%) Show that the conditional probability $P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n \mid Y = y)$ is independent of p.
- 2. (30%) Let X_1, X_2, \dots, X_n be a random sample from $N(0, \sigma^2)$.
 - (a) (10%) Find a sufficient statistic Y for σ^2 .
 - (b) (10%) Show that the maximum likelihood estimator for σ^2 is a function of Y.
 - (c) (10%) Is the maximum likelihood estimator for σ^2 is unbiased?
- 3. (20%) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$. Then
 - (a) (10%) Construct a 95% confidence interval for μ , when the variance σ^2 is known;
 - (b) (10%) Construct a 95% confidence interval for μ , when the variance σ^2 is unknown.
- 4. (30%) Let p_1 and p_2 be the respective proportions of babies with low birth weight (below 2500 grams) of two developed countries. We shall test $H_0: p_1 = p_2$ vs. $H_1: p_1 > p_2$.
 - (a) (10%) Define a critical region that has an $\alpha = 0.05$ significance level.
 - (b) (10%) If respective random samples of size $n_1 = 900$ and $n_2 = 700$ yielded $y_1 = 135$ and $y_2 = 77$ babies with a low birth weight, what is your conclusion?
 - (c) (10%) What would your decision be with a significance level of $\alpha = 0.01$?

(You might need the following information)

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α	0.050	0.025	0.020	0.010	0.005	0.001
Z_{α}	1.645	1.960	2.054	2.326	2.576	3.090
$Z_{\alpha/2}$	1.960	2.240	2.326	2.576	2.807	3.291