

# 淡江大學 98 學年度碩士班招生考試試題

24

系別：數學學系

科目：統計學

准帶項目請打「V」	
✓	簡單型計算機

本試題共 1 頁，4 大題

1. (20%) Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with p.d.f.  $f(x; p) = p^x(1-p)^{1-x}, x = 0, 1$ .  
Then
  - (a) (10%) Show that  $Y = X_1 + X_2 + \dots + X_n$  is a sufficient statistics for  $p$ .
  - (b) (10%) Show that the conditional probability  $P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n | Y = y)$  is independent of  $p$ .
  
2. (30%) Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(0, \sigma^2)$ .
  - (a) (10%) Find a sufficient statistic  $Y$  for  $\sigma^2$ .
  - (b) (10%) Show that the maximum likelihood estimator for  $\sigma^2$  is a function of  $Y$ .
  - (c) (10%) Is the maximum likelihood estimator for  $\sigma^2$  is unbiased?
  
3. (20%) Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(\mu, \sigma^2)$ . Then
  - (a) (10%) Construct a 95% confidence interval for  $\mu$ , when the variance  $\sigma^2$  is known;
  - (b) (10%) Construct a 95% confidence interval for  $\mu$ , when the variance  $\sigma^2$  is unknown.
  
4. (30%) Let  $p_1$  and  $p_2$  be the respective proportions of babies with low birth weight (below 2500 grams) of two developed countries. We shall test  $H_0: p_1 = p_2$  vs.  $H_1: p_1 > p_2$ .
  - (a) (10%) Define a critical region that has an  $\alpha = 0.05$  significance level.
  - (b) (10%) If respective random samples of size  $n_1 = 900$  and  $n_2 = 700$  yielded  $y_1 = 135$  and  $y_2 = 77$  babies with a low birth weight, what is your conclusion?
  - (c) (10%) What would your decision be with a significance level of  $\alpha = 0.01$ ?

(You might need the following information)

$\alpha$	0.050	0.025	0.020	0.010	0.005	0.001
$z_\alpha$	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	1.960	2.240	2.326	2.576	2.807	3.291