

淡江大學 97 學年度碩士班招生考試試題

系別：數學學系

科目：統計學

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁，5 大題

1. (15 points) A random sample X_1, X_2, \dots, X_n from a normal distribution with $\sigma = 1$ is to be used to test the null hypothesis $\mu = \mu_0$ against the alternative hypothesis $\mu = \mu_1$, where $\mu_1 > \mu_0$. Use the Neyman-Pearson lemma to find the best critical region of size α .

2. (15 points) Let X_1, X_2, X_3 have a continuous joint distribution with a density

$$f(x_1, x_2, x_3) = \begin{cases} q(ax_1 + bx_2 + cx_3) & \text{for } 0 \leq x_i \leq 1, i = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

where a, b, c are positive constants. Find:

- q as a function of (a, b, c) .
- The conditional density of (X_1, X_2) given $X_3 = x_3$.
- The conditional density of X_3 given $X_1 = x_1, X_2 = x_2$.

3. (20 points) Is the statistic $Y = (X_1 + 2X_2 + 3X_3)/6$ sufficient for estimating the parameter θ of a Bernoulli population?

4. (20 points) Let $Y_{(1)} < Y_{(2)}$ denote the order statistics of a random sample of size 2 from a normal distribution with mean 0 and variance σ^2 . Find $E(Y_{(1)})$.

5. (30 points) Let α, β, γ be arbitrary positive constants and let U and V be independent chi-square random variables with m and n degrees of freedom, respectively. Let $Z = \gamma U/V$ and $W = \alpha U + \beta V$.

- Find the conditional probability density function of W given $Z = z$.
- Find the distribution of T where T is defined as

$$T = \frac{(\gamma + Z)W}{\alpha Z + \beta \gamma}$$