27-1

淡江大學96學年度碩士班招生考試試題

系別：數學學系　　科目：統　計　學

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

NOTE:

(a) Show your work step by step to receive credit.

(b) You might need the following information: If a r.v. Z has the standard normal distribution, $P\{0 < Z < 2.06\} = .4803$, $P\{0 < Z < 2.17\} = .4850$, $P\{0 < Z < 2.33\} = .4901$, $P\{0 < Z < 2.58\} = .4951$.

1. **(20 points)** Let X, Y be independent, each with the exponential distribution with density $f(x) = \lambda e^{-\lambda x}$. Show that $Z_1 = X + Y$ and $Z_2 = X/(X + Y)$ are independent.

2. **(20 points)** Let $X_1, X_2, \ldots, X_n$ be a random sample from the beta distribution with density $f(x) = Cx^2(1 - x)$, $0 < x < 1$. Let $S_n = X_1 + X_2 + \cdots + X_n$. Find the smallest n for which $P\{S_n \geq 0.75n\} \leq 0.01$.

3. **(20 points)** Conditioning on sample variance $S^2 = s^2$, if r.v. T has a conditional normal distribution $N(0, \sigma^2/s^2)$ and if S^2 is distributed as $(\sigma^2/v)\chi^2_v$, where χ^2_v has the chi-square distribution with v degrees of freedom (df). Prove that T has an unconditional Student's t distribution with v df.

4. **(20 points)** Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with density $f(x; \theta) = (1/\theta)[\theta/(\theta + 1)]^x$, where $x = 1, 2, \ldots$. Determine the UMP test of the hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta > \theta_0$

5. **(20 points)** Let X and Y have the joint density uniform on the triangle with vertices $(0, 0), (2, 0)$ and $(3, 1)$. Find

(a) $E(X|Y)$ and $E(Y|X)$.

(b) $Var(X|Y)$ and $Var(Y|X)$.

(c) $E(X), E(Y), Var(X)$ and $Var(Y)$.