淡江大學 95 學年度碩士班招生考試試題

95

系別：數學學系　　科目：統　計　學

准帶項目請打「V」	
	簡單型計算機

本試題共 一 頁

1. (20%) Let X be one observation from the pdf

$$f(x|\theta) = \left(\frac{\theta}{2}\right)^{|x|}(1-\theta)^{1-|x|},\ x = -1, 0, 1,\ 0 \le \theta \le 1.$$

a) Is X a complete sufficient statistic?
b) Is $|X|$ a complete sufficient statistic?

2. (15%) Let $X_1, \ldots, X_n$ be i.i.d. random variables from the Binomial $(1, \theta)$ distribution, $0 < \theta < 1$. Find a UMVU estimator of the variance of the X's.

3. (30%) Let $X_1, \ldots, X_n$ be i.i.d. with pdf

$$f(x|\theta) = \frac{1}{\theta}, 0 \le x \le \theta, \theta > 0.$$

a) Estimate θ using both the method of moments and maximum likelihood.
b) Calculate the means and variances of the two estimators in Part (a).
c) Which estimator in Part (a) should be preferred and why?

4. (20%) Suppose that $X_1, \ldots, X_n$ are a random sample of size n from a Beta $(\theta, 1)$ distribution, where θ is unknown.
a) Show that the random variable $-2\theta \ln X$ is distributed as χ^2_2.
b) Use Part (a) to construct a confidence interval for θ with confidence coefficient $1 - \alpha$.

5. (15%) Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x\|H_0)$	.01	.01	.01	.01	.01	.01	.94
$f(x\|H_1)$	.06	.05	.04	.03	.02	.01	.79

Use the Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = 0.4$. Compute the probability of Type II Error for this test.