淡江大學九十四學年度碩士班招生考試試題 32-1

系別：數學學系　　　　科目：統　計　學

准帶項目請打「V」	
	簡單型計算機

本試題共 一 頁

注意事項：解題過程必須寫清楚，若只寫答案不予計分。

(1)(30%) Let $X_1, X_2, \ldots, X_n$ be a r.s.(random sample) from a normal distribution with mean μ and variance σ^2, derive(導出) a $100(1-\alpha)\%$ confidence interval for μ under each of the following conditions：(a) σ^2 is known, (b) σ^2 is unknown, and (c) we cannot assume that our random sample comes from a normal distribution, but n is large.

Note：Use symbols to represent quantiles, for example, let z_α stand for the number that satisfies $P(Z > z_\alpha) = \alpha$, where Z is a standard normal random variable.

(2)(20%) If Y has a binomial distribution with parameters n and p, then it is obvious that Y/n is an unbiased estimator of p. To estimate the variance of Y, we generally use $V = n(Y/n)(1-Y/n)$.

(a) Show that V is a biased estimator for the variance of Y.

(b) Modify V slightly to form an unbiased estimator of the variance of Y. (對 V 做輕微修正，使它變成不偏估計量。)

(3)(20%) Let X_1, X_2 be i.i.d. Poisson(λ) random variables. Use the definition(定義) of sufficient statistics to show that

(a) X_1+X_2 is sufficient for λ.

(b) X_1+2X_2 is not sufficient for λ.

(4) (10%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with the following probability function:

$P(X = t_1) = (1-\theta)/2$, $P(X = t_2) = 1/2$, $P(X = t_3) = \theta/2$, $0<\theta<1$. Find the maximum likelihood estimator of θ.

(5)(20%) (a) State the Neyman-Pearson Lemma(敘述內容即可，不用證明).

(b) Find a most powerful size α test of H_0: $X \sim f_0(x)$, where $f_0(x)$ is the standard normal density, against H_1: $X \sim f_1(x)$, where $f_1(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$, based on a sample of size one.