淡江大學九十三學年度碩士班招生考試試題

系別：數學學系　　科目：統　計　學

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簡單型計算機
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本試題共 一 頁

1. (20%) If X is a random variable having the standard normal distribution and $Y = X^2$.

a) Find the covariance of X and Y.

b) Are X and Y independent? Explain.

2. (20%) Consider a random variable X having binomial distribution with the parameters $\theta = 1/3$ and $n = 3$. Find the probability distribution of $Y = (X-1)^4$.

3. (20%) Given a random sample of size n from a normal population with $\mu = 0$, use the Neyman-Pearson lemma to construct the most powerful critical region of size α to test $H_0 : \sigma = \sigma_0$ against $H_a : \sigma = \sigma_1 > \sigma_0$.

4. (20%) Suppose that there are n_1 and n_2 random samples in groups 1 and 2, respectively, $X_1, X_2, \ldots, X_{n_1}$ in group 1, and $Y_1, Y_2, \ldots, Y_{n_2}$ in group 2. Both random variables X_i and Y_i follow the distribution with probability density function $f_i(t) = \lambda_i \exp(-\lambda_i t), i = 1, 2$. Derive a $100(1-\alpha)\%$ confidence interval for the ratio λ_1/λ_2. (Hint: Find the distributions of $\sum X_i, \sum Y_i$ and $\sum X_i / \sum Y_i$.)

5. (20%) A random variable X has a probability density function

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta},$$

where α is a positive integer. Prove that

$$P(X < k) = P(Y > \alpha - 1),$$

where random variable Y has a probability distribution

$$f(y) = \frac{\lambda^y e^{-\lambda}}{y!}, \text{ and } \lambda = k/\beta.$$