淡江大學九十二學年度碩士班招生考試試題

系別：數學學系　　科目：統　計　學

准帶項目請打「○」否則打「×」
簡單型計算機

注意：過程必須寫清楚，只寫答案不計分。

(1) (15%) Let $\bar{X}$ be the mean of a r.s.(random sample) of size n from a Normal(μ,16) distribution. Find the smallest sample size n such that ($\bar{X}$-1, $\bar{X}$+1) is a 0.90 level confidence interval for μ. Given: P(Z<1.28)=0.90, P(Z<1.645)=0.95, P(Z<1.96)=0.975, where Z is a N(0,1) random variable.

(2) (20%) A random sample of size n is taken from a distribution with density

$f(x)=\frac{2x}{\theta^2}$ for $0 \leq x \leq \theta$, and f(x)=0 otherwise.

(a) Find the MLE(maximum likelihood estimator) of the median of the above distribution.

(b) Is this MLE a minimal sufficient statistic? Explain.

(3) (25%) Consider a r.s. $X_1, X_2, \ldots, X_n$ from a Uniform(0,β) distribution. Let Y_n be the largest order statistic of the sample and $\bar{X}$ be the sample mean.

(a) (10%) Show that $\frac{n+1}{n} Y_n$ and $2\bar{X}$ are both unbiased estimators of β.

(b) (10%) Which of these two estimators is more efficient?

(c) (5%) From an intuitive point of view (從直觀角度), which estimator should be more efficient? Explain.

(4) (25%) Let $X_1, X_2, \ldots, X_n$ be a r.s. from Bernoulli distribution with unknown p.

(a) (10%) Use the Neyman-Pearson Lemma to find the critical region of the uniformly most powerful test for testing H_0 : p=1/3 against H_1 : p>1/3.

(b) (10%) Consider the case n=5, and the observed values of $X_1, X_2, \ldots, X_5$ are x_1=1, x_2=1, x_3=1, x_4=0, x_5=1. Calculate the p-value of your test.

(c) (5%) Explain in words (用一般用語解釋) what your p-value means.

(5) (15%) Base on a r.s. of size 1, we want to test the simple null hypothesis that the probability distribution of X is

x	1	2	3	4	5	6	7
f(x)	1/12	1/12	1/12	1/4	1/6	1/6	1/6

against the composite alternative that the probability distribution is

x	1	2	3	4	5	6	7
g(x)	a/3	b/3	c/3	2/3	0	0	0

where a+b+c=1.

Let α=0.25, find the critical region of the likelihood ratio test and calculate P(type II error).