

淡江大學九十一年度碩士班招生考試試題

系別：數學系

科目：統計學

准帶項目請打「○」否則打「×」	
計算機	字典
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注意：要有計算過程，
只寫答案不予計分。

本試題共 / 頁

1. (20%) Let X_1, \dots, X_n be a random sample from uniform $(0, \theta)$, $\mathcal{U}(0, \theta)$, distribution.
Let $M = X_{(n)} = \max_{1 \leq i \leq n} X_i$.

- a) Find the probability density function of M .
b) Use the result in part (a) to find a 95% confidence interval of θ with the form $[M, aM]$.

2. (15%) Let X be a binomial random variable, $\mathcal{B}(n, \theta)$. Suppose that we want to estimate the variance of X , $\text{Var}X$. Consider the estimator $T(X) = n \cdot \frac{X}{n} \cdot \left(1 - \frac{X}{n}\right)$. Is $T(X)$ unbiased?

3. (25%) Let X_1, X_2, X_3 be i.i.d. random variables with density function

$$f(x; \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \quad \theta > 0$$

- a) Find a sufficient statistic, say T , for θ .
b) Find $E(X_1|T)$, where T is the answer you found in part (a).
c) Find the uniformly minimum variance unbiased estimator (UMVUE) of θ .

4. (15%) A random variable X takes values 0, 1, 2, 3 according to one of the 3 distributions shown in the following table. Suppose we have only one observation on X . Find the maximum likelihood estimate of θ .

$f(x \theta)$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$\theta = -1$	0.2	0.3	0.1	0.4
$\theta = 0$	0.5	0.1	0.2	0.2
$\theta = 1$	0.3	0	0.4	0.3

5. (25%) Consider a discrete random variable X with the density function $f(x|\theta)$, where $\theta = 0$ or 1, defined as shown in the following table:

$f(x \theta)$	$x = -2$	$x = -1$	$x = 0$	$x = 1$	$x = 2$
$\theta = 0$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
$\theta = 1$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

For testing the null hypothesis $H_0 : \theta = 0$ against the alternative $H_1 : \theta = 1$ at the level of significance $\alpha = \frac{1}{6}$.

- a) Find the critical region of the likelihood ratio test.
b) Find the power of test in part (a).
c) Find the power of the test that reject $H_0 : \theta = 0$ when $X = 1$.