淡江大學九十學年度碩士班招生考試試題

系別：數學學系

科目：統　計　學

准帶項目請打「○」否則打「x」	
計算機	字典
x	x

本試題共 一 頁

注意: 過程要寫清楚, 只寫答案不記分。

24% (1) Let X be a binomial (n,p) r.v. (random variable). Suppose we know that n is either 2 or 3 and p is either 1/2 or 2/3, and we have only one observation on X.

(a) Calculate P(X=i), i= 0,1,2,3 under all the 4 possible models (n,p)= (2,1/2),(2,2/3),(3,1/2),(3,2/3)

(b) Find the maximum likelihood estimate of (n,p).

14% (2) Consider a r.s.(random sample) of size 2 from the uniform(0,θ) distribution and let R be the range of this sample, then R has the following density

$$f(t) = \begin{cases} \frac{2}{\theta^2}(\theta - t), & 0<t<\theta \\ 0, & \text{otherwise} \end{cases}$$

Use this result to find c so that R<θ<cR is a (1-α)100% confidence interval for θ.

24% (3) Let $X_1,\ldots,X_n$ be a r.s. of size n from Normal(μ,1) distribution.

(a) Show that $\bar{X}^2 - 1/n$ is the minimum variance unbiased estimator of μ^2 and

(b) Find the variance of this estimator.

14% (4) Let X_1, X_2 be a r.s. of size 2 from the population with density

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0<x<1 \\ 0, & \text{otherwise} \end{cases}$$

If the critical region $x_1 x_2 \geq 3/4$ is used to test the null hypothesis θ=1 against the alternative hypothesis θ=2, what is the power of this test at θ=2?

24% (5) Let $X_1,\ldots,X_n$ be a r.s. of size n from a uniform (θ-1, θ+1) distribution.

Let $U=\min(X_1,\ldots,X_n)$ and $V=\max(X_1,\ldots,X_n)$. Find the mean square error of the unbiased estimators (a) $\bar{X}$ (b) $(U+V)/2$ of θ.