

淡江大學八十九學年度碩士班招生考試試題

系列：數學學系

科目：統計學

本試題共 / 頁

1. Let  $X_1, X_2, \dots, X_n$  be a random sample from uniform  $(0, \theta)$ .  
 20% The objective is to estimate  $\theta$ . Suppose that we take  $X_{(n)}$  (the largest element of the sample) and  $\bar{X}$  as estimators of  $\theta$ . Find the mean square errors of  $X_{(n)}$  and  $\bar{X}$ .

(i) State "Rao-Cramér inequality" (5%).

(ii) In a single Bernoulli trial with probability  $\theta^2$ , i.e.  $P(X=1|\theta) = \theta^2$ ,  $P(X=0|\theta) = 1 - \theta^2$ . Find the Fisher information about  $\theta$  in a single observation  $X$ . (15%)

2. Suppose that  $X$  distributed as uniform  $[0, 1]$  if null hypothesis is true and as the exponential ( $\lambda = \frac{1}{2}$ ) if the alternative hypothesis is true. Find the critical region and the power of this test if  $\alpha = 0.05$

3. Consider a gene with two alleles  $A$  and  $a$ . Let the frequency of gene  $A$  in the population be  $\theta$ . Under random mating, the frequencies of individuals of genotypes  $AA, Aa$ , and  $aa$  are  $\theta^2, 2\theta(1-\theta)$ , and  $(1-\theta)^2$ . Of 200 individuals of genotypes, 25 were  $AA$ , 10 were  $Aa$ , and 165 were  $aa$ . Does this result contradict genetic theory? ( $\alpha = 0.05$ , you can set a correct tabulated value)

4. A simple random sampling is a method of selecting  $n$  units out of the  $N$  units (WTR) such that every one of the  $\binom{N}{n}$  distinct samples has an equal chance of being drawn.

(i) Show that the sample mean  $\bar{y}$  is an unbiased estimate of  $\bar{Y}$ .

(ii) Show that the variance of the sample mean is  $\frac{N-n}{N} \frac{\sum_{i=1}^N (y_i - \bar{Y})^2}{N-1}$ .