3/ P1

淡江大學八十八學年度碩士班招生考試試題

系別：數學學系　　科目：數理統計　　本試題共 2 頁

本試題雙面印製

1. Let $X_1, \cdots, X_n$ be independent, with X_i geometrically distributed with parameter θ.
(30%) That is, X_i has density $f(x;\theta) = \theta(1-\theta)^x$, $x = 0, 1, 2, \cdots$; $0 < \theta < 1$.

(a) Find the MLE of $\frac{1-\theta}{\theta}$.

(b) Find the lower bound for the variance of any unbiased estimator of $\frac{1-\theta}{\theta}$.

(c) Is the MLE in (a) an efficient unbiased estimator of $\frac{1-\theta}{\theta}$?

2. Let $X_1, \cdots, X_n$ and $Y_1, \cdots, Y_m$ be respectively two independent random sample from
(20%) $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, where $\mu_1, \sigma_1^2, \mu_2, \sigma_2^2$ are unknown parameters.

(a) Give a $(1-\alpha)$ confidence interval for σ_1^2/σ_2^2.

(b) Give an approximate $(1-\alpha)$ confidence interval for $\mu_1 - \mu_2$.

3. Let $X_1, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ, σ^2 are unknown.
(30%)

(a) Give a complete sufficient statistic.

(b) Find the best unbiased estimator of $\frac{\mu}{\sigma^2}$.

(c) Find the best unbiased estimator of σ.

4. Let the discrete random variable X have the density $f(x;\theta)$, where $\theta \in \{-1, 0, 1\}$.
(20%) Suppose that the density of X is given in the following table :

x	1	2	3	4
$f(x;-1)$	0.53	0.30	0.00	0.17
$f(x;0)$	0.60	0.20	0.10	0.10
$f(x;1)$	0.60	0.22	0.18	0.00

◀注意背面尚有試題▶

淡江大學八十八學年度碩士班招生考試試題

系別：數學學系　　科目：數理統計　　本試題共 2 頁

本試題雙面印製

Consider testing that $H_0: \theta = 0$ against $H_1: \theta \neq 0$.

(a) Find the size 0.2 likelihood ratio test.

(b) Show that the test which rejects H_0 if $X=2$ is a size 0.2 test which is more powerful than the likelihood ratio test.

◀注意背面尚有試題▶