

淡江大學八十七學年度碩士班入學考試試題

系別：數學系

科目：數理統計

本試題共 二 頁

- (1) Let X_1, \dots, X_n be independent, identically distributed (i.i.d) 20% with $X_i \sim \text{Normal}(\theta, \theta^2)$. Find the maximum likelihood estimator of θ .
- (2) Let X_1, \dots, X_n be a random sample (r.s.) of size n from a 20% population with density $f(x) = e^{-(x-\theta)}$, $x > \theta$. Use the Neyman Pearson Lemma to find a level α most powerful test for $H_0: \theta = \theta_0$ versus $H_1: \theta = \theta_1$, $\theta_1 > \theta_0$.
- (3) Let X_1, \dots, X_n be a r.s. from a Poisson distribution with mean λ , 20% find the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda} = P[X_i = 0]$ by calculating the conditional expectation of some unbiased estimator given a sufficient statistic.
- (4) Let $X_1, X_2, \dots, X_n, X_{n+1}$ be a r.s. of size $n+1$, $n > 1$, from 20% a $N(\mu, \sigma^2)$ distribution. Let $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$ and $S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}$. Find the constant C so that the statistic $\frac{C(\bar{X} - X_{n+1})}{S}$ has a t -distribution. (b) If $n=8$, determine k so that $\Pr[\bar{X} - kS < X_9 < \bar{X} + kS] = 0.80$.
- (5) Let $X_{j1}, X_{j2}, \dots, X_{jn_j}$ be a r.s. of size n_j from a $N(\mu_j, \sigma_j^2)$ 20% distribution, $j=1, \dots, k$. And the k samples are independent. We want to test $H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_k^2$ against H_1 : Not all σ_i^2 are equal. Find the likelihood ratio test statistic.

TABLE IV
The *t*-Distribution*

p-2

$$\Pr(T \leq t) = \int_{-\infty}^t \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r} \Gamma(r/2) (1+w^2/r)^{(r+1)/2}} dw$$

$$[\Pr(T \leq -t) = 1 - \Pr(T \leq t)]$$

<i>r</i>	Pr (<i>T</i> ≤ <i>t</i>)				
	0.90	0.95	0.975	0.99	0.995
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

*This table is abridged from Table III of Fisher and Yates; *Statistical Tables for Biological, Agricultural, and Medical Research*, published by Oliver and Boyd, Ltd., Edinburgh, by permission of the authors and publishers.