

淡江大學 104 學年度碩士班招生考試試題

13

系別：數學學系 A 組

科目：微積分 (含高微)

考試日期：3 月 8 日(星期日) 第 2 節

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1 頁

1. (10 points) Find the radius of convergence and interval of convergence of the series,

$$\sum_{n=1}^{\infty} \frac{x^n}{2n-1}.$$

2. (10 points) Show that $\int_0^{\infty} e^{-x^2} dx$ is convergent.

3. (15 points) Let $f_n : [1, 2] \rightarrow \mathbb{R}$ be defined by $f_n(x) = x/(1+x)^n$.

(a) Prove that $\sum_{n=1}^{\infty} f_n(x)$ is convergent for $x \in [1, 2]$.

(b) Is it uniformly convergent?

(c) Is $\int_1^2 (\sum_{n=1}^{\infty} f_n(x)) dx = \sum_{n=1}^{\infty} \int_1^2 f_n(x) dx$?

4. (15 points) Show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable at $x = 0$, but it is not equal to its Maclaurin series.

5. (10 points) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

6. (15 points) Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

7. (10 points) Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$.

8. (15 points) Let M be a complete metric space and $f : M \rightarrow M$ be a contraction. Show that f has a unique fixed point.