淡江大學 103 學年度碩士班招生考試試題

系別：數學學系　　　　科目：微積分（含高微）

考試日期：3 月 2 日(星期日) 第 2 節　　　　本試題共 10 大題， 1 頁

每題 10 分

1. Find the following limits:

   (a) $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$

   (b) $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

2. Find the slope of the tangent line to the curve $x^2+xy-y^2=1$ at (2,3).
3. Evaluate $\int e^x \sin x\, dx$.
4. Test the following series for convergence

   (a) $\sum_{n=1}^{\infty}\frac{1}{n^2+4}$

   (b) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

5. Evaluate

$$\int_0^1\int_y^1 x^2 e^{xy}\, dx\, dy$$

6. Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty}\frac{(3x-2)^n}{2}.$$

7. Find the derivative

$$\frac{d}{dx}\int_0^{\sqrt{x}}\cos t\, dt$$

8. Use the definition of uniform continuity to show that $f(x)=x^2$ is uniform continuous on [0,1].
9. Let $f_n(x)=\begin{cases} n^2x & 0\le x\le \frac{1}{n}, \\ -n^2x^2+2n & \frac{1}{n}\le x\le \frac{2}{n}, \\ 0 & \frac{2}{n}\le x\le 1. \end{cases}$

   (a) Find $\lim_{n\to\infty} f_n(x)$ for each $x\in[0,1]$.

   (b) Does $f_n(x)$ converge uniformly? Why?

10. Suppose f'' is continuous on (a,b) and $f''>0$ on (a,b). Let $x,y\in(a,b)$, $t\in(0,1)$. Show that $f(tx+(1-t)y)\le tf(x)+(1-t)f(y)$.