#務必書寫過計算程，否則不予計分。

1. Let f and g be bounded real-valued functions on a nonempty set X. Prove that

   $\sup\{f(x)+g(x): x \in X\} \le \sup\{f(x): x \in X\} + \sup\{g(x): x \in X\}$. (10points)

2. Prove that if $\{a_n\}$ is monotone increasing and bounded, then $\{a_n\}$ is convergent.(10points)

3. Let f be defined by $f(x) = x\sin(\frac{1}{x})$ if x >0 and f(x)=0 if x≤0.

   Prove that f is continuous at 0 but f is not differentiable at 0. (15points)

4. Suppose f is continuous on [a, b]. Show that $\int_a^c f(x)\,dx = 0$ for all c in [a, b] if and only if

   f(x) = 0 for all x in [a, b]. (15points)

5. Find the maximum and minimum values of f(x, y, z)= x+2y+3z on the ellipse that is the

   intersection of the cylinder $y^2 + x^2 = 2$ and the plane y+z=1. (15 points)

6. Let $u(x,y)=x^2-y^2$ and v(x,y)=2xy. Compute $\frac{\partial x}{\partial u}$ in term of x and y. (10 points)

7. Let $f_n(x) = n^2(1-x)x^n$, 0≤x≤1.

   (1) Find the limit function f(x) of $f_n(x)$. (5points)

   (2) Evaluate $\int_0^1 f_n(x)\,dx$. (5points)

   (3) Show that $f_n(x)$ is not uniformly convergent on [0,1]. (10 points)

8. Find $\frac{dy}{dx}$ if y= $\int_0^{\cos x} \sqrt{1+t^2}\,dt$.(5 points)